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COHOMOLOGY OF CYCLIC GROUPS OF PRIME SQUARE ORDER

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Let G be a cyclic group of order p^2, p a prime, and let U be its unique proper subgroup. If A is any G-module, then the four cohomology groups

$$H^0(G, A) \quad H^1(G, A) \quad H^0(U, A) \quad H^1(U, A)$$

determine all the cohomology groups of A with respect to G and to U. This article determines what values this ordered set of four groups takes on as A runs through all finitely generated G-modules.

Reduction. Let G be any finite group. A finitely generated G-module M is quotient of a finitely generated G-free module L. The kernel K is Z-free, and since the cohomology of L is zero with respect to all subgroups of G, K is a dimension shift of M. The standard dimension shifting module $P = ZG/(S_G)$ is Z-free, so $K \otimes P$ is a Z-free G-module having the same cohomology as M with respect to all subgroups of G.

PROPOSITION 1. If G is any finite p-group and M any Z-free G-module, the cohomology of M is that of $R \otimes M$ where R is the ring of p-adic integers.

Proof. Because M is Z-free, $0 \to M \to R \otimes M \to R/Z \otimes M \to 0$ is a G-exact sequence. $R/Z \otimes M$ is divisible and p-torsion free, so its cohomology is zero, and $M \to R \otimes M$ induces isomorphism on all cohomology groups.

If M is Z-free and finitely generated, $R \otimes M$ is an R-torsion free, finitely generated RG-module. So we see that if G is any finite p-group, every finitely generated G-module has the same cohomology as a finitely generated, R-torsion free RG-module.

2. Exact sequences. Let G be generated by an element g of order p^2 and let U be its subgroup of order p. Heller and Reiner [2] have determined all indecomposable finitely generated R-torsion free RG-modules:

(a) R with trivial action
(b) $B = R(\omega)$, ω a primitive pth root of 1, $g\omega^j = \omega^{j+1}$
(c) $C = R(\theta)$, θ a primitive p^2th root of 1, $g\theta^j = \theta^{j+1}$

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