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THE LACK OF SELF-ADJOINTNESS IN THREE-POINT BOUNDARY VALUE PROBLEMS

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Suppose that $a < c < b$, $C_{[a,b]}$ is the set of all real-valued continuous functions on $[a, b]$, each of p and q is in $C_{[a,b]}$, $p(x) > 0$ for all x in $[a, b]$ and each of P, Q and S is a real 2×2 matrix. The assumption is made that the only member f of $C_{[a,b]}$ so that $(pf')' - qf = 0$ and

$$(\varDelta) \quad P\begin{bmatrix} f(a) \\ p(a)f'(a) \end{bmatrix} + Q\begin{bmatrix} f(c) \\ p(c)f'(c) \end{bmatrix} + S\begin{bmatrix} f(b) \\ p(b)f'(b) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

is the zero function. It follows that there is a real-valued continuous function K_{12} on $[a, b] \times [a, b]$ such that if g is in $C_{[a,b]}$, then the only element f of $C_{[a,b]}$ so that $(pf')' - qf = g$ and $(\varDelta)$ holds is given by

$$f(x) = \int_a^b K_{12}(x, t)g(t)dt \qquad \text{for all } x \text{ in } [a, b]\,.$$

In this note it is shown that if in addition it is specified that Q is not the zero 2×2 matrix, then K_{12} is not symmetric, i.e., it is not true that $K_{12}(x, t) = K_{12}(t, x)$ for all x, t in $[a, b]$.

The union of (a, c) and (c, b) is denoted by R. The symbol j denotes the identity function on $[a, b]$, i.e., $j(x) = x$ for all x in $[a, b]$. If V is a function from $[a, b] \times [a, b]$ and x is in $[a, b]$, then $V(j, x)$ is the function h such that $h(t) = V(t, x)$ for all t in $[a, b]$. If each of f and $(pf')' - qf$ is in $C_{[a,b]}$, then $(pf')' - qf$ is denoted by Lf.

Given an element g of $C_{[a,b]}$, one has the problem of determining a function f so that

$$(*) \qquad \begin{cases} Lf = g \quad \text{and} \\ (\varDelta) \quad \text{holds}\,. \end{cases}$$

Denote $\begin{bmatrix} 0 & \int_a^t 1/p \\ \int_a^t q & 0 \end{bmatrix}$ by $F(t)$ and $\begin{bmatrix} 0 \\ \int_a^t g \end{bmatrix}$ by $G(t)$ for all t in $[a, b]$.

Then problem $(*)$ may be reformulated as follows: find a function Y from $[a, b]$ to E_2 such that

$$(**) \qquad Y(t) = Y(x) + G(t) - G(x) + \int_x^t dF \cdot Y \quad \text{for all } t, x \text{ in } [a, b] \text{ and}$$