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FURTHER RESULTS IN THE THEORY OF MONODIFFRIC FUNCTIONS

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This paper considers two fundamental problems in the theory of monodiffric functions; i.e., discrete functions which satisfy the partial difference equation

$$f(z+1)-f(z)=-i[f(z+i)-f(z)]$$

on some region of the discrete z-plane, $z=m+in$, $m=0$, ± 1, $\pm 2, \cdots$, $n=0$, ± 1, $\pm 2, \cdots$, and which, accordingly, are analogs of analytic functions.

The first problem considered centers about a process analogous to multiplication. A method of analytic extension is presented whereby a function defined along the real axis may be uniquely extended into the upper-half plane as a monodiffric function. The generalized product of two monodiffric functions may then be defined as the extension of a suitable product on the real axis. This definition is shown to be consistent with prior results.

The second problem is concerned with an analog to the Cauchy integral based upon a discrete singularity function which tends to zero as $|z|$ becomes large. The desired singularity function is obtained and the analogous integral formula presented.

Introduction and notation. In recent years, considerable attention has been given to the development of discrete and semi-discrete analogs of analytic functions (see, for example, references [1] through [4], [6]). The theory of these discrete or semi-discrete functions is obtained by deriving satisfactory analogs of classic results in the theory of analytic functions as well as finding results which have no direct analog in the classic theory.

The discrete analogs of analytic functions have been called by several names. Duffin [1] calls them discrete analytic functions, Ferrand [2] calls them preholomorphic functions, and Isaacs [3, 4] calls them monodiffric functions. In this paper we shall be using the definitions given by Isaacs which differ from those given by Duffin and Ferrand and, accordingly, we shall refer to these discrete analogs as monodiffric functions.

One central problem which has not been completely answered is the analog of a closed multiplication. That is, a procedure which is analogous to multiplication such that the "product" of two monodiffric