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TOEPLITZ FORMS AND ULTRASPHERICAL POLYNOMIALS

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For a fixed $\nu > 0$ we set

$$2^n\left(\nu + \frac{1}{2}\right)_n W_\nu(n, x) = (-1)^n(1 - x^2)^{-\nu+1/2}\left(\frac{d}{dx}\right)^n[(1 - x^2)^{n+\nu-1/2}]$$

where $\left(\nu + \frac{1}{2}\right)_n = \Gamma\left(\nu + \frac{1}{2} + n\right)\Big/\Gamma\left(\nu + \frac{1}{2}\right)$. **The** $W_\nu(n, x)$ **are the ultraspherical polynomials of index** ν **normalized so that** $W_\nu(n, 1) = 1$. **If**

$$\Omega_\nu(dx) = (1 - x^2)^{\nu-1/2}dx\ , \qquad \omega_\nu(n) = \frac{\Gamma(\nu)(n + \nu)\Gamma(n + 2\nu)}{\pi^{1/2}\Gamma\left(\nu + \frac{1}{2}\right)\Gamma(2\nu)n!}$$

then the $W_\nu(n, x)$ **satisfy the orthogonality relations**

$$\int_{-1}^{1} W_\nu(n, x)W_\nu(m, x)\Omega_\nu(dx) = (\omega_\nu(n))^{-1}\delta_{n,m}\ .$$

Because

$$W_\nu(n, x)W_\nu(m, x) = \sum_{k=0}^{\infty} c_\nu(m, n, k)W_\nu(k, x)\omega_\nu(k)$$

where the $c_\nu(m, n, k)$ **are nonnegative, the** $[W_\nu(n, x)]_{n=0}^{\infty}$ **behave rather like characters on a compact group. Consequently certain portions of harmonic analysis, which do not extend to orthogonal polynomials in general, have interesting analogues for ultraspherical polynomials.**

In the present paper this fact is exploited to study the moments of the eigenvalues of generalized Toeplitz matrices constructed using ultraspherical polynomials.

Statement of results. Since we will always work with a fixed ν we will drop the subscript and write

$$W_\nu(n, x) = W(n, x)\ , \quad \Omega_\nu(dx) = \Omega(dx)\ , \quad \omega_\nu(n) = \omega(n)\ .$$

For $f(x) \in L^1(\Omega)$ we set

$$f(x) \sim \sum_{j=0}^{\infty} b(j)\,W(j, x) \tag{1}$$

if

$$b(j) = \omega(j)\int_{-1}^{1} f(x)\,W(j, x)\Omega(dx)\ . \tag{2}$$