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IMAGES OF MEASURABLE SETS

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For a finitely additive and countably multiplicative family H, Measurable H is the family of all sets which are measurable by every Carathéodory outer measure by which the members of H are measurable and complements of members of H are approximable from within. A relation contained in a topological product space is subvalent, if for some countable ordinal α, each horizontal section of the relation has an empty derived set of order α. A topological space is Borelcompact if it and the difference of any two of its closed compact subsets are countable unions of closed compact sets.

It is shown that if X and Y are Borelcompact, Hausdorff spaces with countable bases and R is an analytic and subvalent subset of the cartesian product of X with Y, then the direct R-image of A is Measurable $\mathfrak{F}(Y)$ whenever A is Measurable $\mathfrak{F}(X)$. ($\mathfrak{F}(X)$ is the family of closed subsets of X.) If X and Y are complete, separable, metric spaces and R is an analytic and subvalent subset of $X \times Y$, the same conclusion can be drawn.

In a topological setting, the notion of measurability employed (Definitions 3.4.7 and 3.4.8) comprehends measurability by every Carathéodory outer measure by which closed sets are measurable and open sets can be approximated from within. More particularly if $\mathfrak{F}$ is the family of all real closed sets, then (3.4.8) Measurable $\mathfrak{F}$ is such a family that all real analytic sets belong to it and its members are Lebesgue measurable.

Because a topological setting sufficient for the current theory of analytic sets is required, the spaces concerned are either Borelcompact (Definition 4.13) Hausdorff spaces satisfying the second axiom of countability or complete, separable, metric spaces. Under these restrictions Souslin sets (Definition 3.2) and analytic sets (Definition 3.3) are the same, and a relation which is both Souslin and subvalent is the union of a countable family of relations which preserve measurability. This property of the component relations is obtained in Theorems 5.13 and 5.14. The decomposition of a subvalent relation (Theorem 7.6) is described by a transfinite operation of extraction (Definition 6.1) which is related to the familiar transfinite set derivation in topological spaces. The results announced in the introductory passage are contained in Theorems 8.3 and 8.4.