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A NOTE ON THE CLASS GROUP

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The main result yields some information on the class group of a domain R in terms of the class group of R/xR. With slightly stronger hypotheses than are strictly necessary, we state the main result: Let R be a regular domain, x a prime element contained in the radical of R, and suppose that R/xR is locally a unique factorization domain. Let $\{I_\alpha\}$ be a set of unmixed height 1 ideals of R such that the classes of $\{I_\alpha + xR/xR\}$ generate the class group of R/xR; then the classes of $\{I_\alpha\}$ generate the class group of R.

The result of Samuel's and Buchsbaum's stating that if R is a regular U.F.D., then $R[[X]]$ is a regular U.F.D. [4] has been generalized by P. Salmon and the present author in two different directions. Salmon [2, Prop. 3] showed that if R is a regular domain, x is a prime element of R which is contained in the radical of R, and R/xR is a U.F.D., then R is a U.F.D. It was shown [1, Cor. 4] that the map of the class group of R into the class group of $R[[X]]$ is onto if R is a regular noetherian domain. We have found a theorem which simultaneously generalizes the last two results, and even allows a little weakening of the hypotheses.

To set the notation and terminology, we will say that a domain R is locally U.F.D. if the quotient ring R_M is a U.F.D. for all maximal ideals M of R. For any Krull domain R, we will denote the class group (see [3]) of R by $C(R)$. If I is an unmixed height 1 ideal of a Krull domain R, we will denote the class of the class group determined by I by $\mathrm{cl}(I)$. Finally, all rings considered will be commutative noetherian domains with identity.

We wish to capitalize on a simple description of the class group valid for domains which are locally U.F.D. We do so and prepare for the main theorem by a sequence of (probably all known) lemmas.

LEMMA 1. *If R is locally U.F.D., then R is a Krull domain.*

Proof. Since R is noetherian, it is sufficient to show that R is integrally closed. Since $R = \cap R_M$ as M runs over all maximal ideals of R, it will be enough to see that each R_M is integrally closed. But each R_M is a U.F.D., hence integrally closed.

LEMMA 2. *If R is locally U.F.D. and P is a height* 1 *prime of R, then P is invertible.*