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EVERY ABELIAN GROUP IS A CLASS GROUP

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Let T be the set of minimal primes of a Krull domain A. If S is a subset of T, we form $B = \cap A_P$ for $P \in S$ and study the relation of the class group of B to that of A. We find that the class group of B is always a homomorphic image of that of A. We use this type of construction to obtain a Krull domain with specified class group and then alter such a Krull domain to obtain a Dedekind domain with the same class group.

Let A be a Krull domain with quotient field K. Thus A is an intersection of rank 1 discrete valuation rings; and if $x \in K$, x is a unit in all but a finite number of these valuation rings. If P is a minimal prime ideal of A, then A_P is a rank 1 discrete valuation ring and must occur in any intersection displaying A as a Krull domain. In fact, if T denotes the set of minimal prime ideals of A, then $A = \bigcap_{P \in T} A_P$ displays A as a Krull domain.

Choose a subset S of T ($S \neq \emptyset$) and form the domain $B = \bigcap_{P \in S} A_p$. It is immediate that B is also a Krull domain which contains A and has quotient field K. If one of the A_P were eliminable from the intersection representing B, it would also be eliminable from that representing A. Thus the A_P for $P \in S$ are exactly the rings of the type B_Q, where Q is a minimal prime ideal of B. If Q is minimal prime ideal of B, then $Q \cap A = P$ for the $P \in S$ such that $B_Q = A_P$.

Let A and B be generic labels throughout this paper for a Krull domain A and a Krull domain B formed from A as above. We recall that the valuation rings A_P are called the essential valuation rings, and we will denote by V_P the valuation of A going with A_P. We summarize and add a complement to the above.

PROPOSITION 1. *With A and B as above, B is a Krull domain containing A, and the A_P for $P \in S$ are the essential valuation rings of B. Every ring B is of the form A_M for some multiplicative set M if and only if the class group of A is torsion.*

Proof. Everything in the first assertion has been given above.

Suppose the class group of A is torsion; then for each Q_i in $T - S$ choose an integer n_i such that $Q_i^{(n_i)}$ is principal, say $Q_i^{(n_i)} = As_i$. Let M be the multiplicative set generated by all s_i. Then [3, 33.5,