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A NOTE ON LOOPS

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An Associative Element of a quasigroup is defined to be an element a with the property that $x(yz) = a$ implies $(xy)z = a$.

It is then shown that

(i) a quasigroup which contains an associative element is a loop,

(ii) if a loop contains an associative element then the nuclei coincide,

(iii) if a loop is weak inverse then the set of associative elements coincides with the nucleus,

(iv) if a loop is not weak inverse then no associative element is a member of the nucleus and the product of any two associative elements is not associative.

In [2] Osborn defines a Weak Inverse Loop to be a loop with the property that $x(yz) = 1$ implies $(xy)z = 1$. More generally we will define an *Associative Element* of a quasigroup to be an element a with the property that $x(yz) = a$ implies $(xy)z = a$. In this note some of the properties of associative elements will be considered.

LEMMA 1. *If a is an associative element of a quasigroup G then $(xy)z = a$ implies $x(yz) = a$.*

Proof. Assume that $(xy)z = a$. Since G is a quasigroup there exists an element v such that $v(yz) = a$. Hence, since a is associative $(vy)z = a$. Thus $(vy)z = (xy)z$ and so $x = v$ since G is a quasi-group.

THEOREM 2. *A quasigroup which contains an associative element is a loop.*

Proof. Let a be an associative element and y any element of the quasigroup, then there exist elements z and b such that $(ay)z = a$ and $ba = a$. Thus $a = b[(ay)z] = [b(ay)]z$, since a is associative. But $a = (ay)z$ and so $b(ay) = ay$. However y is any element of the quasigroup and so $bx = x$ for all x in the quasigroup. Thus b is a left unit and similarly there exists a right unit and hence a unit element.

Not all loops contain associative elements, for example the loop given by the following multiplication table.