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QUASIMEASURES AND OPERATORS COMMUTING WITH CONVOLUTION

G. I. GAUDRY

Let G be a Hausdorff locally compact abelian group. In this paper we characterise completely those continuous linear operators T from $C_c(G)$ (the space of continuous functions with compact supports endowed with the inductive limit topology) into $M(G)$ (the space of measures with the vague topology of measures) which commute with convolution: $T(f*g) = (Tf)*g$. They are represented by convolution with a "quasimeasure". As a corollary of this theorem, we have the result that the space of multipliers from $L^p(G)(p \neq \infty)$ to $L^q(G)$ is isomorphic to a subspace of the space of quasimeasures.

The quasimeasures are defined as the elements of the dual of a certain inductive limit of Banach spaces. We develop some of the theory of pseudomeasures and of quasimeasures and establish the structural relationship of quasimeasures to pseudomeasures.

Throughout, G will denote a Hausdorff locally compact abelian group, X its character group. M, M_{bd}, M_c will denote the spaces of measures, bounded measures and measures with compact supports respectively. Where necessary, we shall write $M(G), M(X)$ etc. to distinguish the spaces of measures etc. over G and X. We shall write ε_a for the Dirac measure at the point a.

Several function spaces will be of importance:

C will be the space of continuous complex-valued functions. C_c with denote the space of continuous functions with compact supports, regarded topologically as the internal inductive limit of the spaces $C_{c,K}$ (the space of continuous functions with support in K, K compact, and the usual sup norm topology). The support of a function $f \in C$ will be denoted $[f]$.

$L^p(1 \leqq p \leqq \infty)$ will be the Lebesgue spaces determined by Haar measure, the elements being equivalence classes as usual. The Haar measures $dx, d\chi$ on G, X respectively will be assumed normalised so that Plancherel's Theorem holds.

$A(G)$ will denote the space of Fourier transforms of functions integrable over X. By virtue of the semi-simplicity of $L^1(X), A(G)$ is isomorphic to $L^1(X)$. We define the topology of $A(G)$ so as to make it a Banach algebra under pointwise multiplication as follows:

$$\|\hat{f}\|_{A(G)} = \|f\|_{L^1(X)} \qquad (\hat{f} \in A(G))$$

$A_c(G)$ is the subspace of $A(G)$ formed of functions whose supports are