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SECOND ORDER DISSIPATIVE OPERATORS

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A theory of dissipative operators has been developed and successfully applied by R. S. Phillips to the Cauchy problem for hyperbolic and parabolic systems of linear partial differential equations with time invariant coefficients. Our purpose is to show that the Cauchy problem for another system of equations can be brought within the scope of this theory. For this system of equations, we shall parallel the early work of Phillips on dissipative hyperbolic systems. This system of equations is general enough to include, as special cases, such equations as the one dimensional Schrödinger equation and the fourth order equation describing the damped vibrations of a rod.

Several of the results necessary to accomplish this task provide generalizations of the work of A. R. Sims on secondary conditions for nonselfadjoint second order ordinary differential operators.

The system of linear partial differential equations which we consider is of the following form

$$(1.1)\qquad y_t = (Ay_x)_x + (By)_x + Cy$$

where $t \geqq 0$ and $x \in J = (a, b)$, an open subinterval (possibly improper) of the real line. The coefficients are assumed to be time invariant, that is to say, they depend only on the variable x. Several other conditions are imposed on the coefficients. Specifically, for each $x \in J$, $A(x)$ is nonsingular and skew-hermitian, $B(x)$ is hermitian and

$$(1.2)\qquad D(x) \equiv [B_x + C + C^*](x)$$

is nonpositive under the inner product $(y, z) = \sum y^i \bar{z}^i$ in E^m (complex Euclidean m-space). This last condition is referred to as a dissipative condition and in associated physical models reflects a supposition of no energy sources in the interior of the system.

Besides these conditions the elements of A and B are required to be absolutely continuous on compact subintervals of J, and their derivatives and the elements of C are required to be square integrable on such subintervals. Attention is called to the fact that no conditions are imposed on the coefficients at the ends of the interval J.

Before proceeding it will be convenient to announce some conventions with regard to notation. Let $F(x) = I - D(x)$. The symbol H_1 will be used to denote the Hilbert Space $L^2(a, b, F)$ with weight F. Since $F(x) \geqq I$ we may introduce the Hilbert space $H_2 = L^2(a, b, F^{-1})$.