PACIFIC JOURNAL OF MATHEMATICS
Vol. 18, No. 3, 1966

EMBEDDING THEOREMS FOR COMMUTATIVE BANACH ALGEBRAS

WILLIAM G. BADE AND PHILIP C. CURTIS, JR.

One knows from the Gelfand theory that every commutative semisimple Banach algebra $\mathfrak{A}$ containing an identity is a separating subalgebra of the algebra of all complex continuous functions on the space of maximal ideals of $\mathfrak{A}$. We shall be concerned in this paper with conditions which when imposed on a separating Banach subalgebra $\mathfrak{A}$ of $C(\Omega)$, Ω a compact Hausdorff space, will guarantee that $\mathfrak{A} = C(\Omega)$. The conditions will take the form of restrictions on either the algebra or the space Ω. For example we prove that if $\mathfrak{A}$ is an ε-normal Banach subalgebra of $C(\Omega)$ then $\mathfrak{A} = C(\Omega)$ if an appropriate boundedness condition holds locally on Ω. If Ω is assumed to be an F space in the sense of Gillman and Henriksen this boundedness assumption is redundant. These results include a recent characterization of Sidon sets in discrete groups due to Rudin and have applications to interpolation problems for bounded analytic functions.

Various conditions which guarantee that $\mathfrak{A} = C(\Omega)$ are known. One, due to Glicksberg [5], is the following.

(1) Assume $\mathfrak{A}$ is sup-norm closed, contains the constants, and in addition assume that the restriction of $\mathfrak{A}$ to each closed subset F of Ω is a closed subalgebra of $C(F)$.

Another, due to the present authors [1], is the following:

(2) Assume Ω is a totally disconnected F-space and that Ω is the maximal ideal space for $\mathfrak{A}$.

A compact space Ω is an F-space if disjoint open F_σ sets in Ω have disjoint closures. This class of spaces was introduced by Gillman and Henriksen in [4] and includes stonian and σ-stonian spaces as well as their closed subsets. There are also connected examples.

The results in this paper center around extensions of these conditions as well as others due to Katznelson [11, 12]. Many of the techniques apply equally well in a Banach space setting, and are discussed in this way where possible.

To begin the discussion we need the following definition: given $\varepsilon > 0$, call a subset $\mathfrak{F} \subset C(\Omega)$ an *ε-normal family* if for each pair F_1, F_2 of disjoint compact subsets of Ω there exists an $x \in \mathfrak{F}$ satisfying

(i) $|x(\omega) - 1| < \varepsilon, \qquad \omega \in F_1,$

(ii) $|x(\omega)| < \varepsilon, \qquad \omega \in F_2.$

By a *Banach subalgebra* of $C(\Omega)$ we will mean a subalgebra of $C(\Omega)$, not necessarily containing the constants, which is a Banach algebra