PACIFIC JOURNAL OF MATHEMATICS
Vol. 19, No. 1, 1966

ON THE ESSENTIAL SPECTRUM OF THE HYDROGEN ENERGY AND RELATED OPERATORS

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Titchmarsh determined the spectrum of the Schrödinger energy operator associated with the hydrogen atom, i.e. the operator $-\Delta - 1/r$. He showed, in particular, that its essential spectrum consists of the positive real axis. On the other hand, Agudo-Wolf and Birman formulated overlapping criteria for a potential, which ensured that addition of such a potential does not change the essential spectrum of $-\Delta$.

These criteria do not admit the potential $1/r$ and a criterion admitting it is formulated in the forthcoming work of Balslev where he also considers operators in L_p spaces. In this paper we slightly extend this Balslev criterion, in case the operator is a Schrödinger operator. Our proofs are different, inasmuch as we capitalize on the representation of the kernel of the *unperturbed* resolvent. Then we make essential use of a result of Friedrichs which gives a bound for the norm of an integral operator.

A criterion which ensures that the essential spectra of two self adjoint operators are equal is the following: for an appropriate complex number ζ the difference of their resolvents is compact. We shall refer to this property by saying that these operators are resolvent congruent. This property requires less and ensures less, then the one introduced independently by de Branges [18] and Birman-Krein [11] in connection with the perturbation of the continuous spectrum. It is implied by one which we called relative compactness by Gokhberg-Krein [24] and for convenience this is shown in §1. For semi-bounded operators still another property was introduced by Birman [9] who showed that his property also implies this one.

In §1 we recall the classical notion of the essential spectrum, and formulate some general operator theoretic facts, which are used in the subsequent sections. Section 2 contains two theorems. In the first one we show, in particular, that the local square integrability of $1/r$ and the fact that

$$\frac{1}{r} \longrightarrow 0 \quad \text{as} \quad r \longrightarrow \infty ,$$

imply that this potential is compact with reference to Δ. Theorem 2.1 is our basic theorem inasmuch as it is used to establish the ones that follow. Theorem 2.2 shows that the resolvent congruence pro-