PACIFIC JOURNAL OF MATHEMATICS
Vol. 19, No. 1, 1966

SETS OF CONSTANT WIDTH

G. D. CHAKERIAN

A lower bound, better than those previously known, is given for the volume of a 3-dimensional body of constant width 1. Bounds are also given in the case of n-dimensional bodies of constant width 1, $n \geqq 4$. Short proofs of the known sharp bounds for such bodies in the Euclidean and Minkowskian planes are given using properties of mixed areas. An application is made to a measure of outer symmetry of sets of constant width in 2 and 3 dimensions.

Let K be a convex body in n-dimensional Euclidean space E_n. For each point u on the unit sphere S centered at the origin, let $b(u)$ be the distance between the two parallel supporting hyperplanes of K orthogonal to the direction. The function $b(u)$ is the "width function" of K. If $b(u)$ is constant on S, then we say K is a body of constant width.

If K_1 and K_2 are convex bodies, then $K_1 + K_2$ is the "Minkowski sum" or "vector sum" of K_1 and K_2 [5, p. 79]. The following useful theorem is well-known.

THEOREM 1. *A convex body K has constant width b if and only if $K + (-K)$ is a spherical ball of radius b.*

In the case of E_2, a number of special properties of sets of constant width are known-for example, the following theorem of Pàl (see [5, p. 127]).

THEOREM 2. *Any plane convex body B of constant width admits a circumscribed regular hexagon H.*

We shall be concerned with the following type of result, due to Blaschke and Lebesgue (see [1], [3], [4], [5, p. 128], [9]).

THEOREM 3. *Any plane convex body B of constant width 1 has area not less than $(\pi - \sqrt{3})/2$, the area of a Reuleaux triangle of width 1.*

The following short proof of Theorem 3 will set the stage for some later arguments.

Proof of Theorem 3. Let $A(K)$ denote the area of K. The "mixed area" of the plane convex bodies K_1 and K_2, $A(K_1, K_2)$, can be