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TWO THEOREMS ON METRIZABILITY OF MOORE SPACES

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One of the outstanding questions in point set topology is whether each normal Moore space is metrizable. The primary result of this paper is to reduce that question to the problem of deciding whether each normal Moore space is locally metrizable at some point. Also, the question of metrizability for normal, separable spaces is reduced to that for normal, separable, locally compact spaces.

The question as to whether every normal Moore space is metrizable has received considerable attention [1], [2]. In this note it is proved that if every normal Moore space is locally metrizable at some point, then every normal Moore space is metrizable. Also, the question of metrizability for normal, separable spaces is reduced to that for normal, separable locally compact spaces. This complements Jones' result (2, Theorem 5) that every normal separable Moore space is metrizable, provided $2^{\aleph_0} < 2^{\aleph_1}$. In the proof of the first theorem, a construction device similar to one described by Roy [4] is used.

By a development is meant a sequence of collections of regions satisfying Axiom 0 and the first three parts of Axiom 1 of [3].

THEOREM 1. *If there is a normal Moore space which is not metrizable, there is one which is not locally metrizable at any point.*

Proof. Suppose that S^0 is a nonmetrizable, normal Moore space, M^0 is a dense subset of S^0 which is of minimal cardinality, and $\{G_n^0\}$ is a development of S^0. (By taking M^0 to be of minimal cardinality, a property such as separability of S^0 is preserved.) There exist sequences $w = S^0, S^1, S^2, \cdots, u = M^0, M^1, M^2, \cdots,$ and $v = \{G_n^0\}, \{G_n^1\}, \{G_n^2\}, \cdots$ such that for each integer $j > 0$

(i) M^j is a dense subset of $S^j - S^{j-1}$ of minimal cardinality,

(ii) for each point P of M^j and each positive integer k,

$$S_{P,k}^{j+1} \text{ is } S^0 \times (P) \times (k) \text{ and } S^{j+1} \text{ is } S^j + \sum_{P \in M^j} \sum_{k=1}^{\infty} S_{P,k}^{j+1},$$

(iii) the statement that R^{j+1} is a region of G^{j+1} means that either

(1) for some point P of M^j and some region R^0 of G_n^0 and some positive integer i, R^{j+1} is $R^0 \times (P) \times (i)$, or

(2) for some point P of M^j and some region R^j of G_n^j containing P and some positive integer $i \geqq n$,