PACIFIC JOURNAL OF MATHEMATICS
Vol. 19, No. 2, 1966

RESTRICTED BIPARTITE PARTITIONS

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Let $\pi_k(n, m)$ denote the number of partitions

$$n = n_1 + n_2 + \cdots + n_k$$
$$m = m_1 + m_2 + \cdots + m_k$$

subject to the conditions

$$\min(n_j, m_j) \geqq \max(n_{j+1}, m_{j+1}) \qquad (j = 1, 2, \cdots, k-1) .$$

Put

$$\xi^{(k)}(x, y) = \sum_{n,m=0}^{\infty} \pi_k(n, m) x^n y^m .$$

We show that

$$\xi^{(k)}(x, y) = \prod_{j=1}^{k} \frac{1 - x^{2j-1}y^{2j-1}}{(1 - x^j y^j)(1 - x^j y^{j-1})(1 - x^{j-1}y^j)} ,$$

$$\sum_{n,m=0}^{\infty} \pi(n, m; \lambda) x^n y^m = 1 + (1 - \lambda) \sum_{k=1}^{\infty} \lambda^k \xi^{(k)}(x, y) ,$$

$$\sum_{n\, m=0}^{\infty} \phi(n, m) x^n y^m = \sum_{n=0}^{\infty} x^n y^n \xi^{(n)}(x^2, y^2) ,$$

where $\pi(n, m; \lambda)$ denotes the number of "weighted" partitions of (n, m) and $\phi(n, m)$ is the number of partitions into odd parts (n_j, m_j all odd).

Consider partitions of the bipartite (n, m) of the type

$$n = n_1 + n_2 + n_3 + \cdots$$
$$m = m_1 + m_2 + m_3 + \cdots , \tag{1.1}$$

where the n_j, m_j are nonnegative integers subject to the conditions

$$\min(n_j, m_j) \geqq \max(n_{j+1}, m_{j+1}) \qquad (j = 1, 2, 3, \cdots) . \tag{1.2}$$

For brevity we may write (1.2) in the form

$$(n_j, m_j) \geqq (n_{j+1}, m_{j+1}) \qquad (j = 1, 2, 3, \cdots)$$

and say that the "parts" of the partition (1.1) decrease.

Let $\pi(n, m)$ denote the number of partitions (1.1) that satisfy (1.2) and let $\rho(n, m)$ denote the numbers of partitions (1.1) that satisfy

$$(n_j, m_j) > (n_{j+1}, m_{j+1}) \qquad (j=1, 2, 3, \cdots) . \tag{1.3}$$

By the inequality (1.3) is understood