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K- AND L-KERNELS ON AN ARBITRARY RIEMANN SURFACE

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The l-kernel which was first considered by Schiffer for plane regions is extended to arbitrary open Riemann surfaces for a number of significant subspaces of the space of square integrable harmonic differentials Γ_h. The l-kernel for each of the subspaces considered is expressed in terms of the principal functions. Thus if W is an open Riemann surface and p and q the L_1 principal functions of W with singularities Re $1/z$ and Im $1/z$ respectively, then the following result is proved.

THEOREM. **The differential $dp - dq^*$ is an l-kernel for the space Γ_h.**

The l-kernel and another kernel function called the k-kernel by Schiffer are applied to the solution of some well known extremal problems on open Riemann surfaces.

It should be noted here that these problems have also been considered by G. Weill [9]. Finally, the properties of the kernel functions are used to obtain a test for when a surface is of class 0_{AD}.

M. Schiffer in [7] defined the k- and l-kernels for plane regions G. The k-kernel reproduces the value of every square integrable analytic function on G at a prescribed point while the l-kernel is orthogonal to the space of square integrable analytic functions on G with Dirichlet metric. Schiffer showed that these kernel functions can be expressed in terms of the Green's function thus enabling one to actually construct them for a given region.

An important question is whether the k- and l-kernels can be generalized to arbitrary open Riemann surfaces and, if so, whether they can be expressed in terms of functions depending only on the surface as in the case of place regions. We shall answer this question in the affirmative for a number of significant subspaces of the space of square integrable harmonic differentials. In addition, we shall see that these generalized k- and l-kernels have important extremal properties.

1. Principal functions

2. In this section we shall briefly review certain results on principal functions (see cf. [1] pp. 148–186) that will be needed later on.

Let W be the interior of a compact bordered Riemann surface $\bar{W}$