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# LOCALLY TRIVIAL $C^r$ GROUPOIDS AND THEIR REPRESENTATIONS

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**We develop a theory of representations of a locally trivial $C^r$ groupoid, $Z$, on a $C^r$ fiber bundle, $E$ (with fiber $Y$, Lie group $G$, and base space $M=$ the set of units of $Z$).**

**A covariant functor, $A$, is defined, sending $E$ into a locally trivial $C^r$ groupoid $A(E)=$ the groupoid of admissible maps between fibers of $E$, with a natural $C^r$ structure. A $C^r$ bundle map $h: E \to E'$ is sent into a $C^r$ isomorphism $A(h): A(E) \to A(E')$. Properties of the functor $A$ are studied.**

**A $C^r$ representation of $Z$ on $E$ is defined as a $C^r$ homomorphism $\rho: Z \to A(E)$. Let $Z_{ee}$ be the group of elements in $Z$ with $e$ as the left and right unit. We obtain the important result that a $C^r$ homomorphism $\rho_e: Z_{ee} \to A(E_e)$ has an (essentially) unique extension to a $C^r$ representation of $Z$ on a $C^r$ fiber bundle $E'$, where $E'$ is determined by $Z$ and $\rho_e$. This leads to interesting applications in differential geometry. The representations of $L^k$, the (locally trivial $C^\infty$) groupoid of invertible $k$-jets of $C^\infty$ maps of a $C^\infty$ manifold, $M$, into itself, provide (but are not the same as) natural fiber bundles of order $k$ in the sense of Nijenhuis.**

The correspondence obtained in 4.93 and 4.94 between transitive $C^r$ representations of $Z$ and closed subgroups of $Z_{ee}$ is related to the classification of geometric objects for $Z = L^k$ and to the classification of covering spaces for $Z=$ the fundamental groupoid.

In § 5 we discuss some examples-the universal covering groupoid of $Z$, and the fundamental groupoid.

Locally trivial $C^r$ groupoids are as defined in [2], except that we deal only with sets in the usual sense, and our groupoids are always transitive, i.e. groupoids in the sense of Brandt. We find it useful to introduce the notion of a $C^r$ coordinate groupoid in § 2, and show that locally trivial $C^r$ groupoids arise from $C^r$ coordinate groupoids like fiber bundles arise from coordinate bundles.

In § 1 we give a definition for a groupoid which is somewhat more elaborate than the usual one, but it introduces the notation used throughout the paper and it includes some essential properties of groupoids.

1. A groupoid, $Z$ over $M$ (in the sense of Brandt), consists of the following:

1.1. A set, $Z$, with a distinguished subset, $M$, called the set of