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SUB-STATIONARY PROCESSES

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This note supplements the longer paper [3]. It is proved in § 2 that if T is a bounded Schwartz distribution on R^n, e.g. an L^∞ function, then its Fourier transform $\mathscr{F}T$ is of the form $\partial^n f/\partial t_1 \cdots \partial t_n$ where f is integrable over any bounded set to any finite power. This follows from the main theorem of [3], but the proof here is much shorter.

Secondly, § 3 shows that a p-sub-stationary random (Schwartz) distribution has sample distributions of bounded order. This generalizes a result of K. Ito for the stationary case.

Third, in § 4 it is shown that p-sub-stationary stochastic processes define p-sub-stationary random distributions if $p \geqq 1$.

In [5], K. Ito introduced stationary random Schwartz distributions L with second moments. He obtained the "spectral measure" representation of the covariance of L. Using this, he proved for each such L:

(I) There is a finite n such that almost all the sample distributions of L are nth Schwartz derivatives of continuous functions.

The spectral measure also yields

(II) Almost all the sample distributions of L are tempered distributions, and their Fourier transforms are first Schwartz derivatives of locally square-integrable functions.

In [3], (II) was proved for random distributions L which are "p-sub-stationary" for some $p > 1$, i.e. for each f in the Schwartz space $\mathscr{D}$,

$$\sup_h E\,|\,L(\tau_h f)\,|^p < \infty\ ,$$

where $(\tau_h f)(t) = f(t-h)$. Also, "locally square-integrable" was strengthened to "locally integrable to any finite power". In § 2, we shall give corollaries of this result for fixed distributions and stochastic processes with much easier proofs. In § 3, we first prove (I) in the p-sub-stationary case for any $p > 0$, using some lemmas from [3] but no Fourier analysis. Then we obtain a result on the Fourier transform of the covariance for $p = 2$. In § 4, we show that for $p \geqq 1$ a p-sub-stationary stochastic process is also a p-sub-stationary random distribution.