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"THE δ-POINCARÉ ESTIMATE"

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The main application of the theorem proved here is to establish the local solvability of a system of linear partial differential equations, in the analytic case, by a homological procedure based on the associated Spencer resolution and δ-cohomology. The theorem states that the δ-cohomology associated with an involutive system of partial differential equations vanishes in a normed sense. From this one can show that the Spencer resolution associated with an involutive system is exact for analytic data, and thus by a result of D. G. Quillen the corresponding inhomogeneous system has local solutions, provided the inhomogeneous term is analytic and satisfies the appropriate compatibility conditions in the overdetermined case. It is well known that if an arbitrary system is prolonged a sufficient number of times, the resulting system will have vanishing δ-cohomology. According to a result of J. P. Serre this is equivalent to the resulting system being involutive. Thus the question of local solvability reduces to the involutive case, and we obtain the classical existence theorem of Cartan-Kähler.

We prove the theorem only in the case of first order systems. However, there is no loss of generality here because any linear system of partial differential equations can be changed into an equivalent first order system by viewing the lower order derivatives as new variables (see Quillen [3], Prop. 8.2). The theorem proved here was first stated by Spencer [4], who subsequently gave a proof in his paper [5] which, however, is incomplete. Later Ehrenpreis, Guillemin, and Sternberg [1] obtained estimates, by a method different from ours, which are equivalent as far as the above application is concerned. The result of Quillen mentioned above is contained in [3], the result of Serre can be found in [2].

1. The δ-sequence. Let M be a C^∞ manifold of dimension n, and let E and F be vector bundles over M with fiber dimensions m and l. Denote by $\underline{E}$ and $\underline{F}$ the sheaves of germs of C^∞ sections of E and F.

We introduce the jet bundles $J_\mu(E)$ for nonnegative integers μ. The fiber of $J_\mu(E)$ over $q \in M$ is obtained from the stalk $\underline{E}_q$ by indentifying germs which agree up to order μ (in any local coordinate). Coordinates in $J_\mu(E)$ are introduced as follows. Choose a coordinate neighborhood $U \subset M$, with coordinate $x = (x_1, \cdots, x_n)$, over which E