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ENDOMORPHISM RINGS OF PRIMARY ABELIAN GROUPS

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This paper is concerned with the study of certain homomorphic images of the endomorphism rings of primary abelian groups. Let $E(G)$ denote the endomorphism ring of the abelian p-group G, and define $H(G) = \{\alpha \in E(G) \mid x \in G, px = 0$ and height $x < \infty$ imply height $\alpha(x) >$ height $x\}$. Then $H(G)$ is a two sided ideal in $E(G)$ which always contains the Jacobson radical. It is known that the factor ring $E(G)/H(G)$ is naturally isomorphic to a subring R of a direct product $\Pi_{n=1}^{\infty} M_n$ with $\sum_{n=1}^{\infty} M_n$ contained in R and where each M_n is the ring of all linear transformations of a Z_p space whose dimension is equal to the $n-1$ Ulm invarient of G. The main result of this paper provides a partial answer to the unsolved question of which rings R can be realized as $E(G)/H(G)$.

THEOREM. Let R be a countable subring of $\Pi_{\aleph_0} Z_p$ which contains the identity and $\sum_{\aleph_0} Z_p$. Then there exists a p-group G with a standard basic subgroup and containing no elements of infinite height such that $E(G)/H(G)$ is isomorphic to R. Moreover, G can be chosen without proper isomorphic subgroups; in this case, $H(G)$ is the Jacobson radical of $E(G)$.

1. Preliminaries.

(1.1) Throughout this paper p- represents a fixed prime number, N the natural numbers, Z the integers and Z_{p^n} the ring of integers modulo p^n. All groups under consideration will be assumed to be p-primary and abelian. With few exceptions, the notation of [3], [5], and [8] will prevail.

Let $h_G(x)$ and $E(x)$ denote, respectively, the p-height of x in G and the exponential order of x. If A is any subset of the group G, then $\{A\}$ will denote the subgroup of G generated by A. Denote the p^n layer of G by $G[p^n]$. Finally, if A is any set, let $|A|$ be the cardinal number of A.

(1.2) Let G be a p-primary group and B a basic subgroup of G. The group B can be written as $B = \sum_{n \in N} B_n$ where each B_n is a direct sum of, say $f(n)$, copies of Z_{p^n}. That is, $B_n = \sum_{f(n)} \{b_i\}$ where $E(b_i) = n$. Define $H_n = \{p^n G, B_{n+1}, B_{n+2}, \cdots\}$. It is readily verified that $G = B_1 \oplus \cdots \oplus B_n \oplus H_n$ for each $n \in N$. Thus, it is possible to define the projections π_n $(n = 1, 2, \cdots)$ of G onto H_n corresponding to the decomposition $G = B_1 \oplus B_2 \oplus \cdots \oplus B_n \oplus H_n$. Define $\rho_1 = 1 - \pi_1$ and