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MAPPINGS AND SPACES

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Let φ be a closed continuous mapping from X onto Y. It is an open problem whether the realcompactness of X implies the realcompactness of Y. Concerning this problem, in case φ is an open WZ-mapping, we discuss the structure of the image space Y under φ and give a necessary and sufficient condition that Y be realcompact. We also show that if X is locally compact, countably paracompact, normal space then the image space Y of X under a closed mapping is realcompact when X is realcompact.

The notion of realcompact space was introduced by E. Hewitt [7] under the name of Q-spaces. The importance of this notion has been recognized and investigated by many mathematicians (cf. [4, 7]). In this paper we shall discuss the relations between realcompactness and closed continuous mappings and treat also the relations between pseudocompactness and continuous mappings.

As a generalization of closed mappings[1], we have a Z-mapping. Here we shall introduce the notion of WZ-mappings as a further generalization of closed mappings. In Theorem 2.1, we shall prove that pseudocompactness of a space X is equivalent to any one of the following conditions: 1) any continuous mapping from X onto any weakly separable space is always a Z-mapping, (2) the projection: $Y \times X \rightarrow Y$ is a Z-mapping for any weakly separable space Y. We denote by $\varphi: X \rightarrow Y$ a mapping φ from X onto Y; then φ can be extended to a continuous mapping $\Phi: \beta X \rightarrow \beta Y$, called the *Stone extension* of φ, where βX and βY are the Stone Čech compactifications of X and Y resp. (In the sequel we denote always by Φ the Stone extension of φ). In § 4, we shall deal with an extension of an open mapping, and show, in Theorem 4.4, that if $\varphi: X \rightarrow Y$ is a WZ-mapping, then Φ is open if and only if φ is open. This plays an important role in § 6. We shall consider in § 5 the inverse images of realcompact space under Z-mappings. It is known that if φ is a mapping from a given space X onto a realcompact space Y, then $\Phi^{-1}(Y)$ is realcompact [4, p. 148]. In Theorem 5.3, we shall show

[1] Throughout this paper we assume that all our spaces are completely regular T_1-spaces and mappings are continuous. We use, in the sequel. the same notations as in [4]. For instance, $C(X)$ is the set of all continuous functions defined on X. A subset F of X is said to be a *zero set* if $F = \{x; f(x) = 0\}$ (briefly, $F = Z(f) = Z_X(f)$) for some $f \in C(X)$. Cl_A denotes a closure operation in a space A.