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DEDEKIND GROUPS

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A Dedekind group is a group in which every subgroup is normal. The author gives two characterizations of such groups, one in terms of sequences of subgroups and the other in terms of factors in the group.

In a 1940 paper by R. Baer [1], two characterizations of the class of Dedekind groups are given. However, Lemma 5.2 of that paper is in error. The results which follow Lemma 5.2 depend on the validity of that Lemma. Hence these results, which constitute Baer's characterizations of Dedekind groups, are also in error. A portion of this paper is devoted to new characterizations of Dedekind groups which are similar to those attempted by Baer. In §7 there are some further related results.

2. **Notation.** As usual $o(G)$ and $Z(G)$ will denote the order of G and the center of G, respectively. If S is a subset of a group G, then $\langle S\rangle$ is defined to be the subgroup generated by S. G' will denote the commutator subgroup of G. If H is a subgroup of G, then Core (H) is the maximum subgroup of H which is normal in G.

3. **Definitions.** Let $G_1(p, n)$, where p is a prime and n is a nonnegative integer, be defined as the group generated by the two elements a and b, subject to the following relations:

(1) b and $c = b^{-1}a^{-1}ba$ are both of order p,

(2) $ac = ca$, $bc = cb$, $a^p = c^n$.

It is noted that $G_1(p, n)$ is isomorphic to $G_1(p, n')$ if neither n nor n' is 0. Also, for $p = 2$, $G_1(p, n)$ is the dihedral group of order 8, and for $p > 2$, $G_1(p, 0)$ and $G_1(p, 1)$ are the two non-Abelian groups of order p^3[2, pp. 51–52].

Let $G_k(p)$, for $k > 1$ and p a prime, denote the group which is generated by the two elements a and b, subject to the following relations:

$$a^{p^{k+1}} = e,\ b^p = e,\ ba = a^{1+p^k}b\ .$$

Then $o(G_k(p)) = p^{k+2}$ and is the unique group of that order with cyclic subgroup of index p, and a commutator subgroup of order p [2, p. 187].

A group G is said to satisfy condition:

(N_k) if $S \subset T \subseteqq G$, implies $S \lhd T$, whenever the complete lattice of subgroups properly between S and T consists only of a chain of at