PACIFIC JOURNAL OF MATHEMATICS
Vol. 21, No. 2, 1967

REGULAR SEMIGROUPS WHOSE IDEMPOTENTS SATISFY PERMUTATION IDENTITIES

MIYUKI YAMADA

This paper is concerned with a certain class of regular semigroups. It is well-known that a regular semigroup in which the set of idempotents satisfies commutativity $x_1x_2 = x_2x_1$ is an inverse semigroup firstly introduced by V. V. Vagner, and the structure of inverse semigroups was clarified by A. E. Liber, W. D. Munn, G. B. Preston and V. V. Vagner, etc. By a generalized inverse semigroup is meant a regular semigroup in which the set of idempotents satisfies a permutation identity $x_1x_2 \cdots x_n = x_{p_1}x_{p_2} \cdots x_{p_n}$ (where $(p_1, p_2, \cdots, p_n)$ is a nontrivial permutation of $(1, 2, \cdots, n)$). N. Kimura and the author proved in a previous paper that any band B satisfying a permutation identity satisfies normality $x_1x_2x_3x_4 = x_1x_3x_2x_4$. Such a B is called a normal band, and the structure of normal bands was completely determined. In this paper, first a structure theorem for generalized inverse semigroups is established. Next, as a special case, it is proved that a regular semigroup is isomorphic to the spined product (a special subdirect product) of a normal band and a commutative regular semigroup if and only if it satisfies a permutation identity. The problem of classifying all permutation identities on regular semigroups into equivalence classes is also solved. Finally, some theorems are given to clarify the mutual relations between several conditions on semigroups. In particular, it is proved that an inverse semigroup satisfying a permutation identity is necessarily commutative.

A semigroup S is called regular if it satisfies the following:

(1.1) For any element a of S, there exists an element a^* such that $aa^*a = a$.

A *semigroup G admitting relative inverses* introduced by Clifford [1], i.e., a semigroup G satisfying the following condition (1.2) is clearly regular:

(1.2) For any element a of G, there exists an element a^* such that $a^*a = aa^*$ and $aa^*a = a$.

However, the converse is not true. It is well-known that a semigroup is a semigroup admitting relative inverses if and only if it is a union of groups. Consider the symmetric inverse semigroup on the set $\{1, 2\}$ (for definition, see [3], p. 29). Then this semigroup is regular but not a union of groups.