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ON ANTI-AUTOMORPHISMS OF VON NEUMANN ALGEBRAS

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Two types of $*$-anti-automorphisms of a von Neumann algebra $\mathfrak{A}$ acting on a Hilbert space $\mathscr{H}$ leaving the center of $\mathfrak{A}$ elementwise fixed are discussed, those of order two and those of the form $A \to V^{-1}A^*V$, V being a conjugate linear isometry of $\mathscr{H}$ onto itself such that $V^2 \in \mathfrak{A}$. The latter anti-automorphisms are called inner, and are the composition of inner $*$-automorphisms and $*$-anti-automorphisms of the form $A \to JA^*J$, where J is a conjugation, i.e. a conjugate linear isometry of $\mathscr{H}$ onto itself such that $J^2 = I$. The former anti-automorphisms are also closely related to conjugations; they are almost, and in many cases exactly of the form $A \to JA^*J$. Moreover, the existence of $*$-anti-automorphisms of order two leaving the center fixed implies the existence of a conjugation J such that $J\mathfrak{A}J = \mathfrak{A}$, and such that $JA^*J = A$ for all A in the center of $\mathfrak{A}$.

There are two main problems concerning $*$-anti-automorphisms of von Neumann algebras, namely their existence and their description. In the present paper we shall deal with the latter question. It turns out that anti-automorphisms are closely associated with conjugations, a conjugation being a conjugate linear isometry of a Hilbert space onto itself whose square is the identity. This is not surprising, as such maps induce most of the important anti-isomorphisms of von Neumann algebras, cf. [1]. We shall characterize two classes of anti-automorphisms, namely those of order two leaving the center of the von Neumann algebra elementwise fixed, and the so-called inner anti-automorphisms, both characterizations being in terms of conjugations. In the process of doing so we shall make heavy use of Jordan and real operator algebra theory, as developed in [8], [9], and [10]. The second section is devoted to this theory; we shall generalize some of the results in [8] and [9], and in particular classify all weakly closed self-adjoint real abelian operator algebras.

We refer the reader to [1] for terminology and results concerning von Neumann algebras. If $\mathscr{R}$ is a family of operators on a Hilbert space we denote by $\mathscr{R}_{SA}$ the set of self-adjoint operators in $\mathscr{R}$. We say $\mathscr{R}$ is *self-adjoint* if $A^* \in \mathscr{R}$ whenever $A \in \mathscr{R}$. $\mathscr{R}$ is a *self-adjoint real operator algebra* if $\mathscr{R}$ is a self-adjoint family of operators which form an algebra over the real numbers. By a *JW-algebra* we shall mean a weakly closed real linear family of self-adjoint operators closed under squaring. By a *real $*$-isomorphism* of one self-adjoint