

CANONICAL DOMAINS IN SEVERAL COMPLEX VARIABLES

SADAO KATŌ

The main purpose of this paper is to give the function which maps a bounded domain onto the m -representative domain ($m \geq 1$) (hereafter called m -representative function) without utilizing the minimum problems in the case of several complex variables. One of the results obtained is that a m -representative domain becomes also a $(m + 1)$ -representative domain.

Let D be a bounded domain and $k_D(z, \bar{t})$, $z, t \in D$ be the Bergman kernel function. Recently M. Maschler [7] made use of the minimum problems to establish the m -representative function in one variable:

$$(I) \quad w(z) = \frac{M_D^{010 \cdots 0}(z, t_0)}{m_D^{10 \cdots 0}(z, t_0)} \text{ fixed } t_0 \in D,$$

where $M_D^{010 \cdots 0}(z, t_0)$, $m_D^{10 \cdots 0}(z, t_0)$ are both Maschler's minimizing functions and represented in a closed form by using $k_D(z, \bar{t})$ and its derivatives, respectively. Moreover this result has been generalized successfully by T. Tsuboi [13] in the case of several complex variables. In this case, however, for example the 2-representative function of a unit circle is nonregular if we choose a fixed point t_0 in $1/2 < |t_0| < 1$.

In this paper we consider the following m -representative function of other type which coincides with the ordinary Bergman representative function when $m = 1$ (§ 4):

$$(II) \quad w(z) = \int_{t_0}^z N_D^{E_n 0 \cdots 0}(z, t_0) dz,$$

$$N_D^{E_n 0 \cdots 0}(z, t_0) \equiv (E_n 0 \cdots 0) \begin{pmatrix} T_D(t_0, \bar{t}_0) \cdots \frac{\partial^{m-1}}{\partial z^{m-1}} T_D(t_0, \bar{t}_0) \\ \vdots \\ \frac{\partial^{m-1}}{\partial t^{*m-1}} T_D(t_0, \bar{t}_0) \cdots \frac{\partial^{2(m-1)}}{\partial t^{*m-1} \partial z^{m-1}} T_D(t_0, \bar{t}_0) \end{pmatrix}^{-1} \\ \cdot \begin{pmatrix} T_D(z, \bar{t}_0) \\ \vdots \\ \frac{\partial^{m-1}}{\partial t^{*m-1}} T_D(z, \bar{t}_0) \end{pmatrix},$$

where the matrix function $T_D(z, \bar{t})$ (for definition, see § 1) is, as is well known, relatively invariant under any pseudo-conformal mapping. We define in § 1 the relative invariant $T_{2D}(z, \bar{t})$, which plays an