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A UNIQUENESS THEOREM FOR EDGE-CHROMATIC GRAPHS

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Certain of the Ramsey numbers may be evaluated by construction of edge chromatic graphs. The edges of the complete graph on 17 vertices may be coloured in two colours in such a way that no complete subgraph on 4 vertices has all its edges one colour. In this paper it is proved that this colouring is unique.

The complete graph on n vertices will be called an n-clique. The $\binom{n}{2}$ edges of the n-clique are painted with red and blue. For $p, q \geqq 2$, a (p, q)-colouring is a colouring in which there is no red p-clique (no set of p points interjoined by red lines only) and no blue q-clique. A theorem of Ramsey [7] implies the existence of a least integer $M(p, q)$ such that for $n \geqq M$ no (p, q)-colouring of the n-clique exists. Note that by symmetry $M(p, q) = M(q, p)$.

Obviously, $M(2, q) = q$, but apart from this trivial case, few of these Ramsey numbers are known. Greenwood and Gleason [2] have shown that $M(3, 3) = 6$, $M(3, 4) = 9$, $M(3, 5) = 14$, and $M(4, 4) = 18$. These results are obtained by a different method in [3]. The only other known values are $M(3, 6) = 18$ (see [4] or [6]), and $M(3, 7) = 23$ (See [1]).

In order to establish the value of $M(p, q)$, it is necessary to construct a (p, q)-colouring of the $(M-1)$-clique. Proof of the existence of such colourings is accomplished in [2] using finite field residue theory, and by means of "regular colourings" in [3]. The uniqueness of these colourings is also of interest. For in a $(p, q+1)$-colouring all points joined by blue lines to a given point must be (p, q)-coloured. Similarly, in a $(p+1, q)$-colouring all points joined by red lines to a given point must be (p, q)-coloured. When one is discussing the existence of $(p, q+1)$ and $(p+1, q)$-colourings, it is of considerable help to know what (p, q)-colourings are possible.

A $(4, 4)$-colouring of the 17-clique has been constructed in [2] and [3]. Here it will be proved that this $(4, 4)$-colouring is unique.

2. Preliminary results and definitions.

Two edge chromatic graphs G, H are said to be isomorphic if there exists a one-to-one mapping f of the vertices of G onto the vertices of H such that for each vertex pair X, Y in G, edge $f(X)f(Y)$ in H has the same colour as edge XY in G. A (p, q)-colouring G of the n-clique is said to be