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STOLZ ANGLE CONVERGENCE IN METRIC SPACES

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A function f defined on the real line is said to be a Stolz angle limit function if there is a function ϕ defined on the upper half-plane with property that at each point $(x, 0)$ there is a Stolz angle such that the boundary limit of ϕ relative to the Stolz angle exists and is equal to $f(x)$. In this paper the notion of Stolz angle convergence is extended for functions defined on metric spaces.

2. Definitions and notation. Let (X, ρ) be a metric space and let R^+ denote the positive real nuumbers. A set in $X \times R^+$ of the form

$$\{(y, r) \in X \times R^+ : \rho(x, y) \leqq a \cdot r\} ,$$

where a is some positive real number and x is a point in X, is said to be a *Stolz cone with vertex* x. We will denote such a set by $C(x, a)$.

If (X, ρ) is the real line with the usual metric, then a Stolz cone is a Stolz angle in the upper halfplane which has its vertex on the x-axis and which is symmetric about the line $x = c$ if $(c, 0)$ is its vertex.

Let $f : X \rightarrow R$. Then $\omega(x, f)$ denotes the oscillation of f at x, for x in X.

If (X, ρ) is a metric space, we metrize $X \times R^+$ with the metric ρ' defined by

$$\rho'((x, r), (y, s)) = \max \{\rho(x, y), |r - s|\} .$$

3. Continuous extensions. In the first theorem it is shown that a function f in the first Baire class defined on a compact metric pace X can be extended to a continuous function Φ on $X \times R^+$ such that f is the "nontangential" boundary limit of Φ.

THEOREM 1. *If X is a compact metric space and if $f : X \rightarrow R$ is in the first Baire class, then there is a continuous function $\varphi : X \times R^+ \rightarrow R$ such that for each $x \in X$,*

$$\lim \varphi(u, r) = f(x) \qquad \textit{as } (u, r) \rightarrow (x, 0)$$

relative to any Stolz cone $C(x, a)$.

Proof. Let $\{f_n\}$ be a sequence of continuous real-valued functions on X such that $f_n(x) \rightarrow f(x)$ for each x in X. Define $H : X \times (0,1] \rightarrow R$