PACIFIC JOURNAL OF MATHEMATICS
Vol. 22, No. 3, 1967

ON SOME HYPONORMAL OPERATORS

V. ISTRĂŢESCU

Let H be a Hilbert space and T a hyponormal operator ($T^*T - TT^* \geqq 0$). The first result is: if $(T^*)^p T^q$ is a completely continuous operator then T is normal.

Secondly, part we introduce the class of operators on a Banach space which satisfy the condition

$$||x|| = 1 \qquad ||Tx||^2 \leqq ||T^2x||$$

and we prove the following:

1. $\gamma_T = \lim ||T^n||^{1/n} = ||T||$;
2. **if T is defined on Hilbert space and is completely continuous then T is normal.**

In what follows for this section we suppose that T is a hyponormal operator on Hilbert space H.

THEOREM 1.1. *If T is completely continuous then T is normal.*

This is known ([1], [2], [3]).

The main result of this section is as follows.

THEOREM 1.2. *If $T^{*p}T^q$ is completely continuous where p and q are positive integers then T is normal.*

LEMMA. Let $||T|| = 1$. Then in the Hilbert space H there exists a sequence $\{x_n\}$, $||x_n|| = 1$ such that

$$(1) \qquad ||T^*x_n|| \to 1$$

$$(2) \qquad ||T^m x_n|| \to 1 \qquad m = 1, 2, 3, \cdots,$$

$$(3) \qquad ||T^*Tx_n - x_n|| \to 0$$

$$(4) \qquad ||TT^*x_n - x_n|| \to 0$$

$$(5) \qquad ||T^*T^m x_n - T^{m-1}x_n|| \to 0 \qquad m = 1, 2, 3, \cdots.$$

Proof. We observe that (1) $\Rightarrow$ (4) and (2) $\Rightarrow$ (3). Thus it remains to prove (1), (2), and (5).

By definition there exists a sequence $\{x_n\}$, $||x_n|| = 1$ such that

$$||T^*x_n|| \to ||T^*|| = ||T|| = 1 .$$

It is known [3] that for x, $||x|| = 1$