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A GENERALIZATION OF THE WILDER ARCS

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Fox and Harrold first used the words "Wilder arc" to describe a wild arc in euclidean 3-space E^3 which is the union of two tame arcs meeting only in a common endpoint and which is locally peripherally unknotted (L.P.U.) at this point of intersection. Thus there are imposed (a) conditions upon the embeddings of subarcs of the wild arc and (b) conditions upon the manner in which the subarcs meet. The following definition gives only conditions of type (a): An arc $A \subset E^3$ is almost tame if each point of A lies on a tame subarc of A. Clearly, every Wilder arc is almost tame.

The chief result characterizes the set W of points on an almost tame arc at which the arc can fail to be locally tame. In particular, W is shown to be homeomorphic to a closed countable set W' on the unit interval with the property that a point $x \in W'$ either is the first or last point of W' or x has either an immediate predecessor or an immediate successor. Two further results discuss special cases.

LEMMA 0. *If the arc $A \subset E^3$ is almost tame, then A is locally tame at each of its endpoints and hence is* L.P.U. *at these points.*

This lemma follows immediately from the definition of an almost tame arc.

LEMMA 1. *Let C be an uncountable set in the closed unit interval $I = [0, 1]$ in E^1. Then there is a point in C which is a limit point of C both from below and from above.*

This follows from Theorem 6, Chapter 1 of [4].

LEMMA 2. *Let the arc $A \subset E^3$ be almost tame and let W denote the set of points of A at which A is not locally tame. Then W is a closed countable subset of A. Furthermore, if $h: I \to A$ is a homeomorphism, then under the order relation induced on A by h, one has the property that, for each $x \in W$,* (i) *x is the first point of W* or x is the last point of W *or* (ii) *x has either a unique predecessor or a unique successor.*

Proof. The set of points at which A is locally tame is open so W is obviously closed.

Let x be a point of W. By Lemma 0, we have $0 < h^{-1}(x) < 1$. By assumption of almost tameness, x lies on a tame subarc of A.