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# TRANSFORMATIONS ON TENSOR SPACES

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**In this paper we consider those linear transformations from one tensor product of vector spaces to another which carry nonzero decomposable tensors into nonzero decomposable tensors. We obtain a general decomposition theorem for such transformations. If we suppose further that the transformation maps the space into itself then we have a complete structure theorem in the following two cases: (1) the transformation is onto, and (2) the field is algebraically closed and the tensor space is a product of finite dimensional vector spaces. The main results are contained in Theorems 3.5 and 3.8 which state that the transformation $T: U_1 \otimes \cdots \otimes U_n \rightarrow U_1 \otimes \cdots \otimes U_n$ has the form $T(x_1 \otimes \cdots \otimes x_n) = T_1(x_{\pi(1)}) \otimes \cdots \otimes T_n(x_{\pi(n)})$ where $T_i: U_{\pi(i)} \rightarrow U_i$ are nonsingular and $\pi$ is a permutation. Case (2) generalizes a theorem of Marcus and Moyls.**

Let $F$ be a field and $\{U_a: a \in A\}$ be a finite set of vector spaces over $F$. Let $(U, t) = (\otimes(U_a: a \in A), t)$ be a tensor product. Then $U$ is a vector space over $F$, $t: \prod (U_a: a \in A) \longrightarrow U$ is multilinear and, for any vector space $V$ over $F$ and multilinear map $f: \prod (U_a: a \in A) \longrightarrow V$, there is a unique linear transformation $g: U \longrightarrow V$ such that $g \cdot t = f$. The decomposable tensors of $U$ are defined to be the vectors $t(\prod (u_a: a \in A))$, denoted by $\otimes(u_a: a \in A)$, where $u_a \in U_a$ for $a \in A$.

The proofs of the main theorems are based on the purely combinatorial results of the following section.

## 2. Adjacency preserving functions.

In this section we define the adjacency preserving functions and find a decomposition theorem for them.

Let $A$ be a nonempty finite set and for each $a \in A$ let $S_a$ be a nonempty set. If $J$ is a nonempty subset of $A$ we let $p_J$ denote the projection of $\prod (S_a: a \in A)$ onto $\prod (S_a: a \in J)$. If $J = \{a\}$ we write $p_a$ for $p_J$.

For each $J \subseteq A$ we define an equivalence relation, denoted by $\equiv (\text{mod}\, J)$, on $\prod (S_a: a \in A)$ by setting $x \equiv y \pmod J$ if and only if $p_a(x) = p_a(y)$ for all $a \in A \backslash J$. If $X \subseteq \prod (S_a: a \in A)$ is a nonempty subset of equivalent elements relative to $\equiv (\text{mod}\, J)$ then we call $X$ a $J$-subset. If $J = \{a\}$ is a singleton we use $a$-subset for $J$-subset. We note the following

2.1. LEMMA. *A subset $X \subseteq \prod (S_a: a \in A)$ is an equivalence class relative to $\equiv (\text{mod}\, J)$ if and only if $p_J(X) = \prod (S_a: a \in J)$ and $p_a(X)$*