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EXISTENCE OF LEVI FACTORS IN CERTAIN ALGEBRAIC GROUPS

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If G is a connected algebraic linear group with unipotent radical U, Borel and Tits define a Levi factor of G to be any connected reductive subgroup L of G such that $G = L.U$ (semidirect product in the sense of algebraic groups). This differs from the usual notion of Levi decomposition in Lie theory but leads to equivalent results at characteristic 0. The existence of Levi factors at characteristic p is problematic, in view of an example of a group having no Levi factor constructed by Chevalley (unpublished). In this note sufficient conditions are given for a Levi factor to exist, based on the structure of the Lie algebra of G.

THEOREM. *Let G be a connected algebraic linear group defined over a field of characteristic $p > 2$, with unipotent radical U. Denote by $\mathfrak{G}$, $\mathfrak{U}$ the respective Lie algebras, and suppose the following conditions are satisfied:*

(a) *$\mathfrak{G} = \mathfrak{L} + \mathfrak{U}$, where $\mathfrak{L}$ is a reductive subalgebra (definition below).*

(b) *If T is a maximal torus of G whose Lie algebra is included in $\mathfrak{L}$, then* Ad *T stabilizes $\mathfrak{L}$ (where* Ad: $G \rightarrow$ Aut $(\mathfrak{G})$ *is the adjoint representation of G).*

(c) *Distinct maximal tori of G have distinct Lie algebras. Then G has a Levi factor L, whose Lie algebra is $\mathfrak{L}$.*

It should be observed that, in the presence of (c), conditions (a) and (b) are *necessary* for the existence of a Levi factor [3, § 11]. Condition (c) is far from necessary, as easy examples show, but it is satisfied in certain cases of interest. In fact (c) is equivalent to the requirement that the Lie algebra of a Cartan subgroup of G be a Cartan subalgebra of $\mathfrak{G}$.

We begin by summarizing some facts [3, § 9, 11] about the Lie algebra $\mathfrak{G}$ of a connected algebraic linear group G defined over a field of characteristic $p > 2$. This restriction on p will be assumed throughout the paper. Using a Jordan decomposition theorem of Borel and Springer [1, Prop. 1.3] we define $\mathfrak{G}$ (or a subalgebra of $\mathfrak{G}$) to be *reductive* if it has no nontrivial nil ideal (= ideal consisting of nilpotent elements). A *maximal torus* of $\mathfrak{G}$ is a subalgebra of maximal dimension consisting of commuting semisimple elements. Then:

(1) The Lie algebra $\mathfrak{U}$ of U is the largest nil ideal of $\mathfrak{G}$. In particular, G is reductive if and only if $\mathfrak{G}$ is reductive.