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INTEGRAL EQUIVALENCE OF VECTORS OVER LOCAL MODULAR LATTICES

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Let F be a local field with characteristic unequal to two, and in which the element 2 is not unitary. Let V be a regular quadratic space over F, L a lattice on V. The group of units of L is the subgroup

$$0(L) = \{\sigma \in 0(V) \mid \sigma L = L\}$$

of the orthogonal group $0(V)$. Two vectors u and v in L are defined to be integrally equivalent if there exists an isometry $\sigma \in 0(L)$ mapping one onto the other. This paper gives necessary and sufficient conditions for integral equivalence of vectors when the underlying lattice L is modular.

A very fundamental theorem in all studies of quadratic forms is the well-known Witt's Theorem. Yet, integral versions of it come scarce. However, there has been some stirring signs of interest and activity of late along this direction. The solution for integral equivalence of vectors would, of course, constitute an one-dimensional integral extension of this classic theorem. Recent works by James [3], Knebusch [4], Rosenzweig [8], Trojan [9], and Wall [10] may be consulted for the few known special cases. Earlier in [2] the author had extended Trojan's unramified modular solution to the special case of the so-called depleted modular lattices over any dyadic local field. This paper removes the restriction to the size of the weight ideal associated with the lattice and thereby completes the solution for arbitrary modular lattices over dyadic local fields.

The technicalities involved when dealing with an arbitrary lattice are substantial and not all of which we have been able to overcome. Here again special cases have been solved and they are included in the author's doctoral dissertation [1].

1. Preliminaries.

We shall freely make use of the results and terminologies of [6]. We do, however, wish to emphasize a few important relevant facts.

The ground field F is a fixed dyadic local field that is a finite (ramified or unramified) extension of the usual 2-adic number field Q_2 (including Q_2). We let $\mathcal{O}$ stand for the ring of integers in F, $\mathcal{U}$ for the group of units, $\mathcal{P}$ for the unique maximal ideal, π for a prime element generating $\mathcal{P}$, ord for the ordinal function, and $|\ \ |$ for the normalized multiplicative valuation in prime spot $\mathcal{P}$. The residue