CHARACTERIZATION OF THE CONTINUOUS IMAGES OF ALL PSEUDO-CIRCLES

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The purpose of this paper is to establish a characterization of the continuous images of all pseudo-circles, and develop techniques which may be used in the further investigation of the mapping properties of pseudo-circles. The principal conclusions drawn in this paper from this characterization are the theorems that every planar circularly chainable continuum is the continuous image of a pseudo-circle and every snake-like continuum is the continuous image of a pseudo-circle.

The class of pseudo-circles may be defined to be the class of hereditarily indecomposable circularly chainable continua such that, if M is a pseudo-circle, then M is the intersection of the sets of points of a sequence $D_1, D_2, D_3, \cdots$ of circular chains having the properties that: (1) D_{i+1} is crooked in D_i, $i = 1, 2, 3, \cdots$, (2) D_{i+1} has unit winding number in D_i, $i = 1, 2, 3, \cdots$ and (3) the mesh of D_i approaches zero as i increases without bound. Thus, every nondegenerate proper subcontinuum of a pseudo-circle is a pseudo-arc and every pseudo-circle can be embedded in the plane. In view of the relationships between pseudo-arcs and pseudo-circles and the fact that pseudo-arcs are known to have important mapping properties, a number of questions have been raised in the literature regarding the mapping characteristics of pseudo-circles.

We now amplify the foregoing statements. In a recent paper [6] the author has established a global characterization of the continuous images of the pseudo-arc [1], [8], [11] which is similar in certain respects to the well known Hahn-Mazurkiewicz characterization of the continuous images of the arc. This result, which was also established independently by A. Lelek [9], constituted an answer to a question raised by R. H. Bing at the Summer Institute on Set Theoretic Topology, 1955 [3]. In addition, this characterization proved to be useful in [6] in showing that there does not exist any local topological property which characterizes the continuous images of the pseudo-arc. Furthermore, in a subsequent paper of this author [7] the characterization of the continuous images of the pseudo-arc was used to establish properties of topological operations on the class of continuous images of all snake-like continua.

The purpose of this present paper is to establish a corresponding characterization of the continuous images of all pseudo-circles [2, p. 48] This characterization will be expressed in a manner which is formally