PACIFIC JOURNAL OF MATHEMATICS
Vol. 23, No. 3, 1967

A STUDY OF MULTIVALUED FUNCTIONS

CARLOS J. R. BORGES

The primary purpose of this study is to determine which topological properties of a space are preserved by multivalued functions. Among other results, the following are proved:

(A) Let $F: X \to Y$ be a perfect map from X onto Y, with $F(x) \neq \emptyset$ for each $x \in X$, where X and Y are T_1-spaces whose diagonals are G_δ-sets. Then X is metrizable (stratifiable) if and only if Y is metrizable (stratifiable)-see Theorem 3.2.

(B) If $F: X \to Y$ is a multivalued Y-compact quotient map from a separable metrizable space X onto a regular first countable space Y with a G_δ-diagonal, then Y is separable metrizable (see Theorem 4.5).

(C) Every (usc-) lsc-function F from a closed subset of a stratifiable space X to a topological space Y admits a (usc-) lsc-extension to all of X (see Theorem 5.2).

Multivalued functions have been extensively studied by Kruse [6], Michael [7; 8], Ponomarev [12; 13; 14], Smithson [15] and Strother [17; 18]. Choquet [2] and Hahn [3] have also considered multivalued functions.

2. Preliminary definitions and results.

Because there are many conflicting terminologies in the theory of multivalued functions, we find it necessary to attempt a terminology of our own, which is a direct extension of the most natural and simple terminology of Michael [10] and includes some of Ponomarev's terminology:

DEFINITION 2.1. For any sets X and Y, $F: X \to Y$ is a multivalued function provided that, for each $x \in X$, $F(x)$ is a subset of $Y(F(x)$ need not be a closed or nonempty set as required by Ponomarev and others).

Clearly, single-valued functions are just special cases of multivalued functions and indeed a multivalued function from X to Y can obviously be thought of as a single-valued function from X to $\mathscr{A}(Y)$—the family of all subsets of Y (including the empty set).

DEFINITION 2.2. Let $F: X \to Y$ be a multivalued function. Then

(a) $F(A) = \bigcup \{F(x) \mid x \in A\}$ for each $A \subset X$,

(b) $F^{-1}(B) = \{x \in X \mid F(x) \cap B \neq \emptyset\}$ for each $B \subset Y$ (clearly F^{-1} is a multivalued function from Y to X).

It is quite easy to construct a multivalued function F from a