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A NECESSARY CONDITION FOR d-POLYHEDRALITY

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A graph is d-polyhedral provided it is isomorphic to the graph of a d-dimensional convex polytope. One of the unsolved problems in the field of convex polytopes is to characterize the d-polyhedral graphs for $d > 3$. There are, however, several necessary conditions known for a graph to be d-polyhedral. In this paper we present a new necessary condition which is not implied by the other conditions but which has two of them as corollaries. We also show how this new condition may be useful in solving problems dealing with ambiguity of d-polyhedral graphs.

2. Preliminary remarks. The 3-polyhedral graphs have been shown by Steinitz [8] to be those that are planar and 3-connected. The known necessary conditions for d-polyhedrality, $d > 3$, are:

(i) *A d-polyhedral graph is d-connected* [1].

(ii) *A d-polyhedral graph contains a subgraph homeomorphic to the complete graph of $d + 1$ vertices* [3, 4, 6].

(iii) *The maximum number of components into which a d-polyhedral graph may be separated by removing $n > d$ vertices is equal to the maximum number $\sigma(d, n)$ of facets of a d-dimensional polytope with n vertices* [7].

Let G be a graph homeomorphic to C_d (the complete graph of d vertices). If φ is the homeomorphism then the images under φ of the vertices of C_d will be called the *principal* vertices of G.

3. The main result.

THEOREM 1. *Given a vertex v of a d-polyhedral graph G, let V be the set of vertices joined to v by edges of G. Then V is contained in a $(d-1)$-polyhedral subgraph G' of $G \sim \{v\}$, and G' contains a homeomorphic image of C_d whose principal vertices lie in the set V.*

Proof. We shall make use of the *pulling process* which is discussed in detail in [2]. We say that a d-polytope P' is obtained from a d-polytope P by *pulling* vertex v of P, when we replace the vertex v by a point v' such that the segment $(v, v']$ does not intersect any $(d-1)$-flat determined by vertices of $P \sim \{v\}$, and then take as P' the convex hull of v' and the remaining vertices of P. It follows from the results in [2] that when v has been pulled to v' all $(d-1)$-