PACIFIC JOURNAL OF MATHEMATICS
Vol. 24, No. 2, 1968

SOME APPLICATIONS OF A PROPERTY OF THE FUNCTOR Ef

MYLES TIERNEY

The functors mapping cone, Cf, and its dual, Ef, whose definitions will be recalled below, seem to have been introduced by Puppe and Nomura in 1958 and 1960 respectively. There, various basic properties of these functors were established. Here we shall prove a "3×3 lemma" for the functor Ef (with an obvious dual for Cf). This will be applied in § 3 to the problem of determining a Postnikov system for Ef in terms of f, and to show that any space having a Postnikov decomposition, and whose homotopy is finitely generated, has a decomposition in which the only $K(\pi, n)$'s appearing have π a finitely generated free abelian group.

We remark at the outset, that the basic properties of Ef and Cf, including the above mentioned 3×3 lemma, can be established in a more functorial manner than is attempted here. Since we are mainly interested in applications to Postnikov systems, we have chosen to proceed in as direct a fashion as possible. We shall study the functors Ef and Cf in a suitable abstract category in a later paper.

We shall consider the category of topological spaces with basepoint x_0 in X. (X, Y) will denote the space of (free) maps topologized by the compact open topology. $(X, Y)^{\cdot}$ will denote the space of basepoint preserving maps. We write $X \wedge Y$ for the smash product of X and Y, i.e. $X \times Y$ with $X \times y_0 \cup x_0 \times Y$ collapsed to the basepoint.

The adjointness relations

$$(X \wedge Y, Z)^{\cdot} \approx (X, (Y, Z)^{\cdot})^{\cdot}$$

and

$$(X \times Y, Z) \approx (X, (Y, Z)) ,$$

where $\approx$ means naturally homeomorphic to, are basic for what follows. They hold when, say, Y is locally compact and regular, X is Hausdorff [2]. In view of the importance of these relations to us, we should probably work in the category of quasi spaces [7], where this is universally valid. Due to the relative unfamiliarity of these notions, however, we will stick with spaces, and simply make sure the relations are valid when used.

Let I be the unit interval, and S^1 the circle. Given a space X with basepoint, we shall abbreviate $(I, X)^{\cdot}$, $(S^1, X)^{\cdot}$, $X \wedge I$, and $X \wedge S^1$ by the more usual PX, ΩX, CX, and SX respectively. We shall write e_X: $PX \rightarrow X$ for the endpoint projection of the space of based paths on