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A STABILITY THEOREM FOR A THIRD ORDER NONLINEAR DIFFERENTIAL EQUATION

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A stability theorem and a corollary are proved for a nonlinear nonautonomous third order differential equation. A remark shows that the results do not hold for the linear case.

THEOREM. **Let $p'(t)$ and $q(t)$ be continuous and $q(t) \geqq 0$, $p(t) < 0$ with $p'(t) \geqq 0$. For any A and B suppose**

$$A + Bt - \int_{t_1}^{t} q(s)ds < 0$$

for large t where $Q(t) = \int_{t_0}^{t} q(s)ds$, then any nonoscillatory solution $x(t)$ of the equation

$$\dddot{x} = p(t)\dot{x} + q(t)x^{2n+1} = 0, n = 1, 2, 3, \cdots ,$$

has the following properties;

$$\operatorname{sgn} x = \operatorname{sgn} \ddot{x}, \neq \operatorname{sgn} \dot{x}, \lim_{t\to\infty} \ddot{x}(t) = \lim_{t\to\infty} \dot{x}(t) = 0, \lim_{t\to\infty} |x(t)| = L \geqq 0 ,$$

and $x(t)\dot{x}(t), \ddot{x}(t)$ are monotone functions.

COROLLARY. **If $q(t) > \in > 0$ for large t, then $\lim_{t\to\infty} x(t) = 0$.**

In this paper, a nonoscillatory solution $x(t)$ of a differential equation is one that is continuable for large t and for which there exists a t_0 such that if $t > t_0$ then $x(t) \neq 0$. Under above conditions on $p(t)$ and $q(t)$ there always exist continuable nonoscillatory solutions of the equation

$$\dddot{x} + p(t)\dot{x} + q(t)x^{2n+1} = 0 . \tag{1}$$

This follows from an exercise in [1] by letting

$$x(t) = y_1(t), \dot{x}(t) = -y_2(t), \ddot{x}(t) = y_3(t) ,$$

so that

$$\begin{aligned} \dot{y}_1 &= -y_2 \\ \dot{y}_2 &= -y_3 \\ \dot{y}_3 &= -[q(t)y_1^{2n+1} - p(t)y_2] . \end{aligned}$$

Equation (1) can then be written as the system $\bar{y}' = -f(t, \bar{y})$ where $f(t, \bar{o}) = \bar{o}$, $f(t, \bar{y})$ continuous for $t \geqq 0, y_1, y_2, y_3, \geqq 0$ and $f_k(t, \bar{y}) \geqq 0$, $k = 1, 2, 3,$ for $y_k > 0$. In fact $\|\bar{y}(0)\|$ may be prescribed.