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EQUIVALENT DECOMPOSITION OF R^3

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If G is any monotone decomposition of R^3, let H_G denote the union of the nondegenerate elements of G, and let P_G denote the projection map from R^3 onto the decomposition space R^3/G associated with G. Suppose that F and G are monotone decompositions of R^3 such that each of $\mathrm{Cl}\,(P_F[H_F])$ and $\mathrm{Cl}\,(P_G[H_G])$ is compact and 0-dimensional. Then F and G are *equivalent* decompositions of R^3 if and only if there is a homeomorphism h from R^3/F onto R^3/G such that

$$h[\mathrm{Cl}\,(P_F[H_F])] = \mathrm{Cl}\,(P_G[H_G])\ .$$

A necessary and sufficient condition for two decompositions to be equivalent is given. It is shown that there is a decomposition with only a countable number of nondegenerate elements which is equivalent to the dogbone decomposition, and several related results are obtained.

By introducing the idea of equivalent decompositions of R^3, we are able to analyze in a precise way, a process that seems quite natural in the study of monotone decompositions of R^3 of the type we are considering. If F is a monotone decomposition of R^3, the stipulation that $\mathrm{Cl}\, P_F[H_F]$ be a compact 0-dimensional set is equivalent to the following condition: There is a sequence $M_1, M_2, M_3, \cdots$ of compact 3-manifolds-with-boundary in R^3 such that for each positive integer j, $M_{j+1} \subset \mathrm{Int}\, M_j$ and g is a nondegenerate element of F if and only if g is a nondegenerate component of $\bigcap_{j=1}^{\infty} M_j$.

A process one finds useful in certain situations is one that involves a sequence $f_1, f_2, f_3, \cdots$ of homeomorphisms from R^3 onto R^3 such that (1) f_1 shrinks or stretches M_1, (2) f_2 agrees with f_1 on $R^3 - M_1$ and shrinks or stretches M_2, (3) f_3 agrees with f_2 on $R^3 - M_2$ and shrinks or stretches M_3, and so on. The "new" decomposition has as its nondegenerate elements the nondegenerate components of

$$f_1[M_1] \cap f_2[M_2] \cap f_3[M_3] \cap \cdots\ .$$

We are able to show that under fairly mild restrictions, there exists such a sequence of homeomorphisms if and only if the original decomposition and the "new" one are equivalent in the sense of this paper.

We indicate some examples that illustrate these concepts. The first two examples give instances of previous applications of the ideas of this paper. The remaining ones are described in detail in the