PACIFIC JOURNAL OF MATHEMATICS
Vol. 24, No. 3, 1968

# POINT-LIKE 0-DIMENSIONAL DECOMPOSITIONS OF $S^3$

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**This paper is concerned with upper semicontinuous decompositions of the 3-sphere which have the property that the closure of the sum of the nondegenerate elements projects onto a set which is 0-dimensional in the decomposition space. It is shown that such a decomposition is definable by cubes with handles if it is point-like. This fact is then used to obtain some properties of point-like decompositions of the 3-sphere which imply that the decomposition space is a topological 3-sphere. It is also shown that decompositions of the 3-sphere which are definable by cubes with one hole must be point-like if the decomposition space is a 3-sphere.**

In this paper we consider upper semicontinuous decompositions of $S^3$, the Euclidean 3-sphere. In particular, we shall restrict ourselves to those decompositions $G$ of $S^3$ which have the property that the union of the nondegenerate elements of $G$ projects onto a set whose closure is 0-dimensional in the decomposition space of $G$. We shall refer to such decompositions as 0-dimensional decompositions of $S^3$. Numerous examples of such decompositions appear in the literature. (One should note that some of the examples and results to which we refer are in $E^3$, Euclidean 3-space, but the corresponding examples and results for $S^3$ will be obvious in each case.)

In § 3, a technique of McMillan [10] is used to show that point-like 0-dimensional decompositions of $S^3$ are definable by cubes with handles. Armentrout [2] has shown this in the case where the decomposition space is homeomorphic with $S^3$. The proof of this theorem shows that compact proper subsets of $S^3$ with point-like components are definable by cubes with handles.

In § 4 we give some properties of point-like 0-dimensional decompositions of $S^3$ which imply that the decomposition space is homeomorphic with $S^3$. These properties were suggested by Bing in § 7 of [6].

It is not known whether monotone 0-dimensional decompositions of $S^3$ which yield $S^3$ must have point-like elements. Partial results in this direction have been obtained by Armentrout [2], Bean [5], and Martin [9]. Bing, in § 4 of [6], has presented an example of a decomposition of $S^3$ which yields $S^3$ even though it is not a point-like decomposition, but this example is not 0-dimensional. In § 5 we show that a 0-dimensional decomposition of $S^3$ that yields $S^3$ must have point-like elements if it is definable by cubes with one hole.