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A CHARACTERIZATION OF GROUPS IN TERMS OF THE DEGREES OF THEIR CHARACTERS II

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In this paper we continue our study of the relationship between the structure of a finite group G and the set of degrees of its irreducible complex characters. The following hypotheses on the degrees are considered: (A) G has r.x. e for some prime p, i.e. all the degrees divide p^e, (B) the degrees are linearly ordered by divisibility and all except 1 are divisible by exactly the same set of primes, (C) G has a.c. m, i.e., all the degrees except 1 are equal to some fixed m, (D) all the degrees except 1 are prime (not necessarily the same prime) and (E) all the degrees except 1 are divisible by $p^e > p$ but none is divisible by p^{e+1}. In each of these situations, group theoretic information is deduced from the character theoretic hypothesis and in several cases complete characterizations are obtained.

In situation (A), the greater complexity which can occur when $e \geqq p$ is explored and a conjecture concerning p-groups with $e < p$ is studied and certain cases of it are proved. Detailed statements are made about groups G satisfying (B) for which the common set of prime divisors of the degrees does not consist of a single prime for which G has a nonabelian $\mathfrak{S}_p$ subgroup. These results are applied to situation (C), groups with a.c. m, and such groups are completely characterized when m is not a prime power corresponding to a nonabelian Sylow subgroup. If $m = p^e$ and an $\mathfrak{S}_p$ of G is nonabelian then it is shown that G must be nilpotent unless $e = 1$ (in which case G has r.x. 1 and has been completely characterized in [2]). This reduces the study of groups with a.c. m to p-groups and it is shown that a p-group G with a.c. p^e must have an abelian normal subgroup of index p^e unless G has class 2 or 3. Further information is obtained about these "special" class 2 and 3 groups. It is also shown that if $e > 1$ then G must have class $\leqq p$.

Groups satisfying hypothesis (D) are completely characterized and it is shown that in this case there are at most two degrees different from 1. Finally it is shown that if G satisfies hypothesis (E) and has a nonabelian $\mathfrak{S}_p$ subgroup then G is nilpotent and has a.c. p^e. In all the situations considered in this paper, the group in question is shown to be solvable.

We use here the notation and terminology of [2].

1. Groups with r.x. $(p-1)$. In [4] we classified all groups with