PACIFIC JOURNAL OF MATHEMATICS
Vol. 24, No. 3, 1968

A NOTE ON LEFT MULTIPLICATION OF SEMIGROUP GENERATORS

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It is shown in this note that if A is the infinitesimal generator of a strongly continuous semigroup of contraction operators in any Banach space X, then so is BA for a broad class of bounded operators B; the only requirement on B is that it transforms "in the right direction".

In the recent paper [1] the following interesting result was obtained.

THEOREM 1 (*Dorroh*). *Let X be the Banach space of bounded functions on a set S under the supremum norm, let A be the infinitesimal generator of a contraction semigroup in X, and let B be the operator given by multiplication by p, $pX \subseteqq X$, where p is a positive function defined on S, bounded above, and bounded below above zero. Then BA is also the infinitesimal generator of a contraction semigroup in X.*

This leads naturally to the general question of preservation of the generator property under left multiplication; the purpose of this note is to present Theorem 2 below, which shows that for any Banach space, a large class of operators B are acceptable. In the following, the word "generator" will always mean generator of contraction semigroup.

In this note we will consider only left multiplication by everywhere defined bounded operators B. It is easily seen (e.g., [2, Corollary 3]) that A generates a contraction semigroup if and only if cA does, $c > 0$. Also by [4, Th. 2.1], if A is bounded, BA is a generator if and only if BA is dissipative; in this case clearly right multiplication also yields a generator. See [4, 5] for dissipativeness; we use dissipativeness in the sense [4], and recall that if BA is a generator, then BA is dissipative in all semi-inner products on X.

THEOREM 2. *Let X be any Banach space, A the infinitesimal generator of a contraction semigroup in X, and B a bounded operator in X such that $\|\varepsilon B - I\| < 1$ for some $\varepsilon > 0$. Then BA generates a contraction semigroup in X if and only if BA is dissipative, (i.e., $\mathrm{Re}\,[BAx, x] \leqq 0$, all $x \in D(A)$, $[u, v]$ a semi-inner product (see [4])).*