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PRODUCTS AND QUOTIENTS OF PROBABILISTIC METRIC SPACES

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In this paper some results concerning the products and quotients of probabilistic metric spaces are presented.

Probabilistic metric spaces were first introduced by K. Menger in 1942 and reconsidered by him in the early 1950's [3, 4, 5]. Since 1958, B. Schweizer and A. Sklar have been studying these spaces, and have developed their theory in depth [9, 10, 11, 12, 13]. These spaces have also been considered by several other authors [e. g., 2, 14, 15, 16]. An extensive, detailed up-to-date presentation may be found in [7].

In the sequel, we shall adopt the usual terminology, notation and conventions of the theory of probabilistic metric spaces, with but one exception: In all previous work, the distribution functions which determine the distances between points were required to have supremum one. Our investigations have led us to drop this requirement and the results which we present here show that doing so is natural. It is easy but tedious to check that the restriction to distribution functions with supremum one is not required in any of the previously established results which will be needed in the sequel.

In concluding this introduction we remark that products of probabilistic metric spaces have previously been considered by V. Istratescu and I. Vaduva [2]. However, their definition of Cartesian product employs associative functions which are stronger than Min, the strongest possible triangular norm. Because of this, and in view of the discussion given in [10], their results appear somewhat restrictive. Also, a number of the results concerning finite products, which are presented in § 1 and which were announced in [1], have recently been obtained independently by A. Xavier [17].

1. Product spaces.

DEFINITION 1. Let $(S_1, \mathfrak{F}_1)$ and $(S_2, \mathfrak{F}_2)$ be *PM* spaces and let T be a left-continuous t-norm. The *T-product* $(S_1, \mathfrak{F}_1) \times (S_2, \mathfrak{F}_2)$ of $(S_1, \mathfrak{F}_1)$ and $(S_2, \mathfrak{F}_2)$ is the space $(S_1 \times S_2, T(\mathfrak{F}_1, \mathfrak{F}_2))$, where $S_1 \times S_2$ is the Cartesian product of the sets S_1 and S_2 and $T(\mathfrak{F}_1, \mathfrak{F}_2)$ is the mapping from $(S_1 \times S_2) \times (S_1 \times S_2)$ into the set of distribution functions Δ given by