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POINTLIKE SUBSETS OF A MANIFOLD

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Morton Brown introduced the concept of a cellular subset of S^n. As a consequence of the generalized Schoenflies Theorem it is easy to show that a subset of S^n is pointlike if and only if it is cellular. In this paper the obvious generalization of the definitions of pointlike and cellular sets are made and thier relationship in a manifold is considered. It is easy to show that a cellular subset of a manifold is pointlike. While it is not true that a pointlike subset of a manifold is cellular, it is shown that a pointlike subset of a compact n-manifold lies in a contractible n-manifold with $(n-1)$-sphere boundary. As a consequence of this it is shown that K is a pointlike subset of a compact n-manifold $(n \neq 4)$ if and only if K is cellular. The case $n = 4$ is still unsolved.

DEFINITIONS. An *n-manifold* is a connected separable locally Euclidean metric space. A connected separable metric space in which every point has a neighborhood whose closure is an n-cell is an *n-manifold with boundary.* Note that a manifold is a manifold with boundary boundary but not conversely. A compact connected subset K of an n-manifold M is *pointlike* if $M \sim K$ is homeomorphic with $M \sim \{p\}$ where $p \in M$. A subset K of an n-manifold M is *cellular* if there is a sequence of n-cells $C_1, C_2, \cdots$ such that $C_{i+1} \subset \text{Int } C_i$ and $K = \cap C_i$. An $(n-1)$-sphere S^{n-1} that separates an n-manifold M into components A and B is *collared on the side containing* A if there is an embedding $h: S^{n-1}X[0, 1] \rightarrow \bar{A}$ such that $h(x, 0) = x$. An $(n-1)$-sphere S^{n-1} in an n-manifold M is *bicollared* if there is an embedding $h: S^{n-1}X[0, 1] \rightarrow M$ such that $h(x, 1/2) = x$. A *pseudo-sphere* is a compact manifold that is a homotopy sphere. A compact contractible n-manifold with boundary is called a pseudo-cell. The Poincare Conjecture—known to be true for $n \neq 3, 4$ [7]—says that a pseudo-sphere is a sphere.

PRELIMINARY THEOREMS. The following theorem follows from the corresponding theorem for E^n which is proved by the same methods as used in [4].

THEOREM 1. *A cellular subset of a manifold is pointlike.*

One might think that a pointlike subset of a manifold is cellular. That this is not the case is shown by the following example.

EXAMPLE 1. Let M be E^3 minus the integers on the positive x-axis, and minus 1-spheres of radius 1/4 centered at the negative