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PRIME RINGS WITH A ONE-SIDED IDEAL SATISFYING A POLYNOMIAL IDENTITY

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It is known that the existence of a nonzero commutative one-sided ideal in a prime ring implies that the whole ring is commutative. Since rings satisfying a polynomial identity are natural generalizations of commutative rings the question arises as to what extent the above mentioned result can be extended to include these generalizations. That is, if R is a prime ring and I a nonzero one-sided ideal which satisfies a polynomial identity does R satisfy a polynomial identity?

This paper initiates an investigation of this problem. A counter example, given later, will show that the answer to the above question may be negative, even when R is a simple primitive ring with nonzero socle. The main theorem of this paper is Theorem 3 which states:

Let R be a prime ring having a nonzero right ideal which satisfies a polynomial identity. Then, a necessary and sufficient condition that R satisfy a polynomial identity is that R have zero right singular ideal and $\hat{R}$, the right quotient ring of R, have at most finitely many orthogonal idempotents.

2. In the following given a ring R, $R^{\Delta}({}^{\Delta}R)$ denotes the right (left) *singular ideal* of R. Thus $R^{\Delta} = \{x \mid x \in R, x^r \in L^{\Delta}(R)\}$ where $L^{\Delta}(R)$ denotes the set of right ideals of R that meet, in a nonzero fashion, all right ideals of R. Similarly for ${}^{\Delta}R$ and ${}^{\Delta}L(R)$.

If Q is a ring such that R is a subring of Q and $qR \cap R \neq 0$ for each $q \in Q$ then Q is called a right quotient ring for R. Moreover if $Q = \{ab^{-1} \mid a, b \in R, b \text{ regular}\}$ then Q is called a classical right quotient ring. Following [2] we say that a ring R is right quotient simple if and only if it has a classical right quotient ring Q with $Q \cong D_n$, D_n a ring of $n \times n$ matrices over a division ring D.

From [4] we know that if R is a prime ring with $R^{\Delta} = 0$ then R has a unique maximal right quotient ring $\hat{R}$ where $\hat{R}$ is a prime regular ring. Moreover, letting $L(R)$ denote the lattice of right ideals of R, there is a mapping $s\colon A \to A^s$ of $L(R)$ which is a closure operation satisfying $0^s = 0$, $(A \cap B)^s = A^s \cap B^s$ and $(x^{-1}A)^s = x^{-1}A^s$. The set $L^s(R)$ of closed ideals of R can be made into a lattice in a natural way and it is shown in [4] that $L^s(R) \cong L^s(\hat{R})$ under the mapping $A \to A \cap R$, $A \in L^s(\hat{R})$. We shall have occasion to use the following realization of $\hat{R}$. Let $E = \bigcup_{A \in L^{\Delta}(R)} \mathrm{Hom}_R(A, R)$. On E