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A RIEMANNIAN SPACE WITH STRICTLY POSITIVE SECTIONAL CURVATURE

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Let M_1 and M_2 be two Riemannian spaces[1] with Riemannian metrics d_1 and d_2 respectively whose sectional curvature is positive constant. We consider the product of the two Riemannian spaces $M_1 \times M_2$, then the Riemannian space $M_1 \times M_2$ has nonnegative sectional curvature with respect to the Riemannian metric $d_1 \times d_2$ but not strictly positive sectional curvature.

We can construct a Riemannian metric on $M_1 \times M_2$ which approaches the Riemannian metric $d_1 \times d_2$ as closely as we wish and which has strictly positive sectional curvature.

Now, our results can be stated as follows. We consider two manifolds $M_1(H_1 - E_1, q_1)$, $M_2(H_2 - E_2, q_2)$ such that each of them has only one chart where H_1, E_1 are the south hemisphere and the equator, respectively, of a k-dimensional sphere $(k \geqq 2)$ and E_2, H_2 are also the south hemisphere and the equator, respectively, of an n-dimensional sphere $(n \geqq 2)$, and q_1, q_2 are special mappings. We also consider on M_1 and M_2 particular Riemannian metrics d_1, d_2, respectively, with positive constant sectional curvature. We obtain a special 1-parameter family of Riemannian metrics $F(t)$ on $M_1 \times M_2$ such that $F(0) = d_1 \times d_2$. We have proved that $\forall P \in M_1 \times M_2$ the derivative of the sectional curvature with respect to the parameter t for $t = 0$ and for any plane of $(M_1 \times M_2)_P$, is strictly positive.

1. Let M_1 be a manifold which consists of one chart $(H_1 - E_1, q_1)$, where H_1, E_1 are the south hemisphere and the equator, respectively, of a k-dimensional sphere $S_1^k (k \geqq 2)$ and the inverse mapping of q_1 is defined as follows

$$q_1^{-1} = \left\{ x^1 = \frac{2u_1}{1 + u_1^2 + \cdots + u_k^2}, \cdots, x^k = \frac{2u_k}{1 + u_1^2 + \cdots + u_k^2}, \right.$$
$$\left. x^{k+1} = \frac{u_1^2 + \cdots + u_k^2 - 1}{1 + u_1^2 + \cdots + u_k^2} \right\}.$$

q_1 maps the open set $H_1 - E_1$ onto the open ball $u_1^2 + \cdots + u_k^2 < 1$.

On the manifold M_1, we take the following Riemannian metric

[1] A Riemannian space is a Riemannian manifold covered with one chart ([**5**], p. 314).