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TRANSITIVE AND FULLY TRANSITIVE PRIMARY ABELIAN GROUPS

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This paper is concerned with transitivity and full transitivity of primary abelian groups. It is well known that countable primary groups and primary groups without elements of infinite height are both transitive and fully transitive. The question of whether all primary groups are transitive or fully transitive was recently answered negatively by C. Megibben. Megibben's examples indicate that $p^{\omega}G$ may be transitive (fully transitive) while G is not transitive (fully transitive). For β an ordinal number, we investigate conditions on a primary group G which will insure that G is transitive (fully transitive) whenever $p^{\beta}G$ is transitive (fully transitive). Specifically, we show that if $G/p^{\beta}G$ is a direct sum of countable groups and $p^{\beta}G$ is fully transitive, then G is fully transitive. The same result is established for transitivity except that β is restricted to be a countable ordinal.

All groups considered in this paper are additively written primary abelian groups for a fixed prime p. For the most part, we follow the notation and terminology of [1]. All topological references will be to the p-adic topology. If G is a p-primary group and if β is an ordinal, we, define subgroups $G[p]$ and $p^{\beta}G$ as follows: $G[p] = \{x \in G \mid px = 0\}$; $pG = \{x \in G \mid x = pg, g \in G\}$, $p^{\beta}G = p(p^{\beta-1}G)$ if $\beta - 1$ exists and

$$p^{\beta}G = \bigcap\nolimits_{\alpha<\beta} p^{\alpha}G$$

if β is a limit ordinal. If x is an element of G and G is reduced we define the generalized height $h_G(x)$ and the generalized Ulm sequence $U_G(x)$ of x by:

$$h_G(x) = \begin{cases} \beta & \text{if } x \neq 0 \text{ and } \beta + 1 \text{ is the first ordinal such that} \\ & x \notin p^{\beta+1}G \\ \infty & \text{if } x = 0 \end{cases}$$

$U_G(x) = (\beta_0, \beta_1, \cdots, \beta_i, \cdots)$ where $\beta_i = h_G(p^i x)$ for each integer i. The generalized Ulm sequences are partially ordered in the obvious term-by-term fashion, that is, $U_G(x) \geqq U_G(y)$ if and only if $h_G(p^i x) \geqq h_G(p^i y)$ for all i. We assume, of course, that $\infty > \beta$ for all ordinals β.

Following Kaplansky [3], we call a reduced p-group G fully transitive (transitive) if for each pair of elements x and y in G with $U_G(x) \geqq U_G(y)(U_G(x) = U_G(y))$ there exists an endomorphism (automorphism) φ of G such that $\varphi(y) = x$. Kaplansky has shown in [3] that