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DEFINING SUBSETS OF E^3 BY CUBES

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This paper is concerned with compact subsets of E^3 which are the intersection of a properly nested sequence of compact 3-manifolds with boundary each of which is the union of a finite collection of pairwise disjoint 3-cells. Such sets are characterized by a property of their complements. Related results are stated in terms of embeddings of compact 0-dimensional sets and upper semicontinuous decompositions of E^3.

Theorem 1 below gives an affirmative answer to a question raised by Štan'ko in [10].

1. Definitions and notation. We use E^3 to denote Euclidean 3-space. In [10], a compact set $K \subset E^3$ is defined to be *cellular-divisible* if there is a sequence $\{M_i\}$ of compact 3-manifolds with boundary such that

(1) if $i = 1, 2, \cdots$, then $M_{i+1} \subset \operatorname{Int} M_i$,

(2) if $i = 1, 2, \cdots$, then M_i is the union of a finite collection of pairwise disjoint topological cubes (3-cells), and

(3) $K = \bigcap_{i=1}^{\infty} M_i$.

We shall use the terminology of [9] and say that such a set is *definable by cubes*. By the approximation theorem for 2-spheres [3] there is no loss of generality in supposing that each M_i in the above definition is polyhedral. If K is a continuum (i.e., compact and connected) and is definable by cubes, then K is said to be *cellular*. If K is compact and 0-dimensional, then K is *tame* (*wild*) if and only if K is (is not) definable by cubes. Tameness in this case is equivalent to the existence of a homeomorphism of E^3 onto itself carrying K into a straight line interval. See [5] or [7].

We use Cl for closure, Bd for boundary, Ext for exterior, and Int for interior. Int may mean "combinatorial interior" or "bounded complementary domain" with context providing the proper interpretation in each case. If K is a subset of E^3 and $\varepsilon > 0$, we use $V(K, \varepsilon)$ to denote the ε-neighborhood of K.

2. Subsets of E^3 which are definable by cubes. The following theorem affirmatively answers question 2 of [10]. An example of Kirkor [8] shows that the hypothesis that J can be separated from K by a 2-sphere cannot be replaced by the weaker hypothesis that J can be shrunk to a point in $E^3 - K$.

THEOREM 1. *Suppose $K \subset E^3$ is compact and fails to separate*