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LINEAR FUNCTIONALS ON ORLICZ SPACES: GENERAL THEORY

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Let Φ be a generalized Young's function and L^Φ the corresponding Orlicz space, on a general measure space. The problem considered here is the characterization of the dual space $(L^\Phi)^*$, in terms of integral representations, without any further restrictions. A complete solution of the problem is presented in this paper. If Φ is continuous and the measure space is sigma finite (or localizable), then a characterization of the second dual $(L^\Phi)^{**}$ is also given. A detailed account of the quotient spaces of L^Φ relative to certain subspaces is presented; and the analysis appears useful in the study of such spaces as the Riesz and Köthe-Toeplitz spaces.

The purpose of this paper is two-fold. First it contains a complete study centering around the singular linear functionals, analyzing certain factor (or quotient) spaces, of the Orlicz spaces. Second, the so-called 'generalized Young's functions' and the associated Orlicz spaces, and their adjoint spaces, are also considered. (Precise definitions will be given later.) The work here is a continuation of [19] and the notation and terminology of that paper will be maintained. However, the theory presented here subsumes [19], and the exposition is essentially self-contained.

If Φ and Ψ are complementary Young's functions (cf. Definition 1 below), let L^Φ and L^Ψ be the corresponding Orlicz spaces on a (not necessarily finite or even localizable) measure space (Ω, Σ, μ) which has only the (nonrestrictive) finite subset property. This latter means that every set of positive μ-measure has a subset of positive finite μ-measure. Then the representation problem for continuous linear functionals on L^Φ is to express them as integrals relative to appropriate additive set functions on Σ. In [1] and [19] certain general integral representations of such elements were obtained when the Young's function Φ and the measure μ satisfy some restrictions. If $\mathscr{M}^\Phi$ is the closed subspace of L^Φ spanned by the μ-step functions then $x^* \in (L^\Phi)^*$, adjoint of L^Φ, is termed *singular* if $x^*(\mathscr{M}^\Phi)=0$, i.e., $x^* \in (\mathscr{M}^\Phi)^\perp$, and it is *absolutely continuous* if there exists a $\chi_E \in \mathscr{M}^\Phi$, and $x^*(\chi_E) \neq 0$, where χ_E is the indicator of $E \in \Sigma$. It is known that $(\mathscr{M}^\Phi)^\perp \neq \{0\}$ if Φ is continuous and grows exponentially fast. In Theorem 2 (and hence 3) of [19] it was announced that every $x^* \in (L^\Phi)^*$ is of the form $x^*(f) = \int_\Omega f dG$ for a certain additive set function G. However, the result was proved only for such x^* that $x^*(\chi_E) \neq 0$ for