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PROJECTIONS IN $\mathscr{L}_1$ AND $\mathscr{L}_\infty$-SPACES

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This paper is devoted to the study of bounded Boolean algebras of projections in the spaces L_1 and L_∞, or more generally the spaces $\mathscr{L}_1$ and $\mathscr{L}_\infty$, of Lindenstrauss and Petczyński.

Our results in $\mathscr{L}_1$ show that such algebras of projections are in every way analogous to algebras of projections in Hilbert space: theorems concerning the common refinement of two (or more) commuting such algebras, or limits of scalar operators, are just as true as in Hilbert space. It is also shown that every cyclic subspace is a direct summand isomorphic to an L_1-space and under the additional assumption of finite multiplicity the Boolean algebra of projections is isomorphic to a subalgebra of multiplications by characteristic functions on some L_1-space.

For $\mathscr{L}_\infty$-spaces, we show that strongly σ-complete bounded Boolean algebras of projections are isomorphic to rather trivial algebras on c_0 and under the additional assumption that the underlying space is a $\mathscr{P}$-space it is proved that such algebras of projections have a finite number of elements.

1. **Preliminaries.** We will start by summarizing here some notations, definitions and results which will be useful in what follows. Most of the results in the subsequent sections are concerned with Boolean algebras of projections. A Boolean algebra of projections $\mathscr{E}$ will be called *complete* if for every family $(E_\alpha) \subset \mathscr{E}$ the projections $\bigvee E_\alpha$ and $\bigwedge E_\alpha$ exist in $\mathscr{E}$ and, moreover

$$(\bigvee E_\alpha)X = \mathrm{clm}\,\{E_\alpha X\}$$
$$(\bigwedge E_\alpha)X = \bigcap E_\alpha X\ .$$

A projection $E \in \mathscr{E}$ will be called *countably decomposable* if every family of disjoint projections in $\mathscr{E}$ bounded by E is at most countable. Bade [2, Lemma 3.1] proved that for every $E \in \mathscr{E}$ there is a family of disjoint countably decomposable projections $E_j \in \mathscr{E}$ such that $E = \bigvee E_j$. For each $x \in X$ the projection $C(x) = \bigwedge \{E \mid E \in \mathscr{E}, Ex = x\}$ will be called the *carrier projection of* x. The *cyclic subspace* $\mathfrak{M}(x)$ spanned by a vector x is $\mathrm{clm}\,\{Ex \mid E \in \mathscr{E}\}$. Bade introduced in [2] the multiplicity function $m(\cdot)$ for a complete Boolean abgebra (B.A.) of projections as follows: if $E \in \mathscr{E}$ is countably decomposable, the *multiplicity* of E, $m(E)$, is the smallest cardinal of a set A of vectors