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# $p$-AUTOMORPHIC $p$-GROUPS AND HOMOGENEOUS ALGEBRAS

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**A $p$-group was called $p$-automorphic by Boen, if its automorphism group is transitive on elements of order $p$. Boen conjectured that if $p$ is odd, then such a $p$-group is abelian. Let $P$ be a nonabelian $p$-automorphic $p$-group, $p$ odd, generated by $n$ elements. Boen proved that $n > 3$, and in joint work with Rothaus and Thompson proved that $n > 5$. Kostrikin then showed that $n > p + 6$, as a corollary of results on homogeneous algebras. In this paper it is shown that $n > 2p + 3$, using Kostrikin's methods, and his proof is somewhat simplified by eliminating special case considerations for small values of $p$.**

The above results and the following terminology may be found in [1], [2], and [4]. Let $A$ be a finite-dimensional algebra over the field $K$, where if $x, y \varepsilon A$ and $\lambda \varepsilon K$, we assume bilinearity and the law $(\lambda x) \circ y = \lambda(x \circ y) = x \circ (\lambda y)$, but associativity is not assumed. Following [4], $A$ is said to be *homogeneous* if the automorphism group $\Gamma$ of $A$ is transitive on $A^{\#} = A - \{0\}$, *anticommutative* if $x \circ y + y \circ x = 0$, and *nil* if all endomorphisms $K_a: x \rightarrow x \circ a$ are nilpotent.

For a fixed odd prime $p$, suppose that $P$ is a nonabelian $p$-automorphic $p$-group with minimal number $n$ of generators. It is shown in [1] that $P$ has a $p$-automorphic quotient group $\bar{P}$ with the same number of generators, where the Frattini subgroup $\Phi(\bar{P})$ is central and is the direct product of $n$ cyclic groups of equal order $p^m$. If we consider $A = \bar{P}/\Phi(\bar{P})$ as a vector space over $GF(p)$, we define a multiplication in $A$ as follows: for $x = a\Phi(\bar{P}), y = b\Phi(\bar{P})$ in $A$, a coset $z = c\Phi(\bar{P})$ is uniquely determined, such that $[a, b] = c^{p^m}$. Define $x \circ y = z$. Then it is clear that $A$ becomes an anticommutative homogeneous algebra, and Theorem 1 of [2] asserts that $A$ is nil.

It is proved in [4] that if $A$ is a finite-dimensional homogeneous algebra with nontrivial multiplication over a field $K$ of characteristic not 2, then $A$ is an anticommutative nil algebra and $K$ is a finite field of $q$ elements, where $q < \dim A - 6$. In this paper we shall prove:

THEOREM. *Let $A$ be a homogeneous anticommutative nil algebra with nontrivial multiplication of dimension $n$ over the field $K$ of $q$ elements, $q$ odd. Then $n > 2q + 3$.*

This result immediately implies the corresponding result for $p$-